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Three Sequences of Special Dio Triple

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Abstract: *This paper concerns with the study of constructing three sequences of Dio - triples (a,b,c) such that in each sequence, the product of any two elements with their sum and added by a polynomial with integer coefficient is a perfect square.*

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I. Introduction

A set of m positive integers $\{a_1, a_2, a_3, ...\}$ is said to have the property $D(n)$, $n \in z-(0)$ if $a_i a_j + a_i + a_j + n$, a perfect square for all $1 \le i \le j \le m$ and such a set is called a Special Dio m – tuples with property $D(n)$. Many mathematicians considered the construction of different formulations of Dio - triples with property $D(n)$, [1-14].

II. Method of Analysis

2.1 sequence i

An attempt is made to form a sequence of Dio - triples $(a,b,c),(b,c,d),(c,d,e),...$, with the property $D(n^2+2n+5)$

Case 1: Let $a = 2n+3$ and $b = 4n+2$

Let c be any non-zero integer.

Consider $c(2n+4)+n^2+4n+8 = p_1^2$ (1)

as well $c(4n+3)+n^2+6n+7 = q_1^2$ (2)

Performing some algebra

$(4n+3)p_1^2 - (2n+4)q_1^2 = 2n^3+3n^2+6n-4$

by the linear transformations

$p_1 = X + (2n+4)T$

$q_1 = X + (4n+3)T$

By substituting $T = 1,$ we have

$X = 3n+4$

and $p_1 = 5n+8$.

From (1), $c = 12n+14$.

Hence (a,b,c) is the Special Dio - triple with the property $D(n^2+2n+5)$.

Case 2: Let $b = 4n+2$ and $c = 12n+14$

Let d be any non-zero integer.

Consider $d(4n+3)+n^2+6n+7 = p_2^2$ (3)

as well $d(12n+15)+n^2+14n+19=q_2^2$ (4)

Performing some algebra

$(12n+15)p_2^2-(4n+3)q_2^2=8n^3+28n^2+56n+48$

by the linear transformations

$p_2=X+(4n+3)T$

$q_2=X+(12n+15)T$

By substituting $T=1$, we have

$X=7n+7$

and $p_2=11n+10$.

From (3), $d=30n+31$.

Hence (b,c,d) is the Special Dio - triple with the property $D(n^2+2n+5)$.

Case 3: Let $c=12n+14$ and $d=30n+31$

Let e be any non-zero integer.

Consider $e(12n+15)+n^2+14n+19=p_3^2$ (5)

as well $e(30n+32)+n^2+32n+36=q_3^2$ (6)

Performing some algebra

$(30n+32)p_3^2-(12n+15)q_3^2=18n^3+53n^2+106n+68$

by the linear transformations

$p_3=X+(12n+15)T$

$q_3=X+(30n+32)T$

By substituting $T=1$, we have

$X=19n+22$

and $p_3=31n+37$.

From (5), $e=80n+90$.

Hence (c,d,e) is the Special Dio - triple with the property $D(n^2+2n+5)$.

In all the above cases, $(a,b,c),(b,c,d),(c,d,e),....$ will form a sequence of Dio - triples. For simplicity and clear understanding , sequence of Dio – triples are exhibited in Table 1 for n=0 to 4.

Table 1 : Examples

n	(a,b,c)	(b,c,d)	(c,d,e)	$D(n^2+2n+5)$
0	$(3,2,14)$	$(2,14,31)$	$(14,31,90)$	$D(5)$
1	$(5,6,26)$	$(6,26,61)$	$(26,61,170)$	$D(8)$
2	$(7,10,38)$	$(10,38,91)$	$(38,91,250)$	$D(13)$
3	$(9,14,50)$	$(14,50,121)$	$(50,121,330)$	$D(20)$
4	$(11,18,62)$	$(18,62,151)$	$(62,151,410)$	$D(29)$

Sequence II

An attempt is made to form a sequence of Dio - triples $(a,b,c),(b,c,d),(c,d,e),...$, with the property $D(4n+5)$

Case 1: Let $a=4n+3$ and $b=4n+2$

Let c be any non-zero integer.

Consider $c(4n+4)+8n+8=p_1^2$ (7)

as well $c(4n+3)+8n+7=q_1^2$ (8)

Performing some algebra

$(4n+3)p_1^2-(4n+4)q_1^2=-4n-4$

by the linear transformations

$p_1=X+(4n+4)T$

$q_1=X+(4n+3)T$

By substituting $T=1$, we have

$X=4n+4$

and $p_1=8n+8$.

From (7), $c=16n+14$.

Hence (a,b,c) is the Special Dio - triple with the property $D(4n+5)$.

Case 2: Let $b=4n+2$ and $c=16n+14$

Let d be any non-zero integer.

Consider $d(4n+3)+8n+7=p_2^2$ (9)

as well $d(16n+15)+20n+19=q_2^2$ (10)

Performing some algebra

$(16n+15)p_2^2-(4n+3)q_2^2=48n^2+96n+48$

by the linear transformations

$p_2=X+(4n+3)T$

$q_2=X+(16n+15)T$

By substituting $T=1$, we have

$X=8n+7$

and $p_2=12n+10$.

From (9), $d=36n+31$.

Hence (b,c,d) is the Special Dio - triple with the property $D(4n+5)$.

Case 3: Let $c=16n+14$ and $d=36n+31$

Let e be any non-zero integer.

Consider $e(16n+15)+20n+19=p_3^2$ (11)

as well $e(36n+32)+40n+36=q_3^2$ (12)

Performing some algebra

$(36n+32)p_3^2-(16n+15)q_3^2=80n^2+148n+68$

by the linear transformations

$p_3=X+(16n+15)T$

$q_3=X+(36n+32)T$

By substituting $T=1$, we have

$X=24n+22$

and $p_3=40n+37$.

From (11), $e=100n+90$.

Hence (c,d,e) is the Special Dio - triple with the property $D(4n+5)$.

In all the above cases, $(a,b,c),(b,c,d),(c,d,e),....$ will form a sequence of Dio - triples. For simplicity and clear understanding , sequence of Dio – triples are exhibited in Table 2 for n=0 to 4.

Table 2 : Examples

n	(a,b,c)	(b,c,d)	(c,d,e)	$D(4n+5)$
0	$(3,2,14)$	$(2,14,31)$	$(14,31,90)$	$D(5)$
1	$(7,6,30)$	$(6,30,67)$	$(30,67,190)$	$D(9)$
2	$(11,10,46)$	$(10,46,103)$	$(46,103,290)$	$D(13)$
3	$(15,14,62)$	$(14,62,139)$	$(62,139,390)$	$D(17)$
4	$(19,18,78)$	$(18,78,175)$	$(78,175,490)$	$D(21)$

I. Sequence III

An attempt is made to form a sequence of Dio - triples $(a,b,c),(b,c,d),(c,d,e),\dots$, with the property $D(k^{2n+2}+1)$

Case 1: Let $a=k^{2n}-1-k^{n+1}$ and $b=k^{2n}-1+k^{n+1}$

Let c be any non-zero integer.

Consider $c(k^{2n}-k^{n+1})+k^{2n}-k^{n+1}+k^{2n+2}=p_1^2$ (13)

as well $c(k^{2n}+k^{n+1})+k^{2n}+k^{n+1}+k^{2n+2}=q_1^2$ (14)

Performing some algebra

$(k^{2n}+k^{n+1})p_1^2-(k^{2n}-k^{n+1})q_1^2=2k^{3n+3}$

by the transformations

$p_1=X+(k^{2n}-k^{n+1})T$

$q_1=X+(k^{2n}+k^{n+1})T$

By substituting $T=1,$ we have

$X=k^{2n}$

and $p_1=2k^{2n}-k^{n+1}$.

From (13), $c=4k^{2n}-1$.

Hence (a,b,c) is the Special Dio - triple with the property $D(k^{2n+2}+1)$.

Case 2: Let $b=k^{2n}-1+k^{n+1}$ and $c=4k^{2n}-1$

Let d be any non-zero integer.

Consider $d(k^{2n}+k^{n+1})+k^{2n}+k^{n+1}+k^{2n+2}=p_2^2$ (15)

as well $d(4k^{2n})+4k^{2n}+k^{2n+2}=q_2^2$ (16)

Performing some algebra

$(4k^{2n})p_2^2-(k^{2n}+k^{n+1})q_2^2=3k^{4n+2}-k^{3n+3}$

by the transformations

$p_2=X+(k^{2n}+k^{n+1})T$

$q_2=X+(4k^{2n})T$

By substituting $T=1,$ we have

$X=2k^{2n}+k^{n+1}$

and $p_2=3k^{2n}+2k^{n+1}$.

From (15), $d=9k^{2n}+3k^{n+1}-1$.

Hence (b,c,d) is the Special Dio - triple with the property $D(k^{2n+2}+1)$.

Case 3: Let $c=4k^{2n}-1$ and $d=9k^{2n}+3k^{n+1}-1$

Let e be any non-zero integer.

Consider $e\left(4k^{2n}\right)+4k^{2n}+k^{2n+2}=p_3^2$ (17)

as well $e\left(9k^{2n}+3k^{n+1}\right)+9k^{2n}+3k^{n+1}+k^{2n+2}=q_3^2$ (18)

Performing some algebra

$$\left(9k^{2n}+3k^{n+1}\right)p_3^2-\left(4k^{2n}\right)q_3^2=5k^{4n+2}+3k^{3n+3}$$

by the transformations

$$p_3=X+\left(4k^{2n}\right)T$$

$$q_3=X+\left(9k^{2n}+3k^{n+1}\right)T$$

By substituting $T=1,$ we have

$$X=6k^{2n}+k^{n+1}$$

and $p_3=10k^{2n}+k^{n+1}.$

From (17), $e=25k^{2n}+5k^{n+1}-1.$

Hence (c,d,e) is the Special Dio - triple with the property $D\left(k^{2n+2}+1\right).$

In all the above cases, $(a,b,c),(b,c,d),(c,d,e),....$ will form a sequence of Dio - triples. For simplicity and clear understanding , sequence of Dio – triples are exhibited in Table 3 for n=0 to 4.

Table 3 : Example**s**

n	(a,b,c)	(b,c,d)	(c,d,e)	$D\left(k^{2n+2}+1\right)$
0	$(-k,k,3)$	$(k,3,3k+8)$	$(3,3k+8,5k+24)$	$D\left(k^2+1\right)$
1	$\left(-1,2k^2-1,4k^2-1\right)$	$\left(2k^2-1,4k^2-1,12k^2-\right.$	$\left(4k^2-1,12k^2-1,30k^2-\right.$	$D\left(k^4+1\right)$
2	$\begin{pmatrix} k^4-1-k^3,k^4-1 \\ +k^3,4k^4-1 \end{pmatrix}$	$\begin{pmatrix} k^4-1+k^3,4k^4-1, \\ 9k^4+3k^3-1 \end{pmatrix}$	$\begin{pmatrix} 4k^4-1,9k^4+3k^3-1, \\ 25k^4+5k^3-1 \end{pmatrix}$	$D\left(k^6+1\right)$
3	$\begin{pmatrix} k^6-1-k^4,k^6-1 \\ +k^4,4k^6-1 \end{pmatrix}$	$\begin{pmatrix} k^6-1+k^4,4k^6-1, \\ 9k^6+3k^4-1 \end{pmatrix}$	$\begin{pmatrix} 4k^6-1,9k^6+3k^4-1, \\ 25k^6+5k^4-1 \end{pmatrix}$	$D\left(k^8+1\right)$
4	$\begin{pmatrix} k^8-1-k^5,k^8-1 \\ +k^5,4k^8-1 \end{pmatrix}$	$\begin{pmatrix} k^8-1+k^5,4k^8-1, \\ 9k^8+3k^5-1 \end{pmatrix}$	$\begin{pmatrix} 4k^8-1,9k^8+3k^5-1, \\ 25k^8+5k^5-1 \end{pmatrix}$	$D\left(k^{10}+1\right)$

II. Conclusion

This paper concerns with the construction of special dio - triples involving three different sequences of triples, that cannot be extended to a quadruple. One may search for special dio-triples consisting of special numbers with suitable property.

References

[1]. Abu Muriefah F.S , Al-Rashed .A, Some Diophantine quadruples in the ring Z[ ] Vol.9 , Math. Commun, 2004,,1-8.

[2]. Assaf.E , Gueron.S, Characterization of regular Diophantine quadruples, Vol.56, Elem. Math.2001,71-81.

[3]. Bashmakova.I.G(ed.), Diophantus of Alexandria, Arithmetic's and the book of polynomial numbers, Nauka Moscow, 1974

[4]. Dujella.A, Some polynomial formulas for Diophantine quadruples, Grazer Math. Ber.328 1996 , 25-30.

[5]. Dujella.A, Some estimates of the number of Diophantine quadruples, Vol.53, Publ. Math. Debrecen ,1998,177-189.

[6]. Filipin.A and Fujita.Y, Any polynomial D(4)-quadruple is regular, Vol.13, Math. Commun. ,2008,45-55.

[7]. [7]. Filipin.A, An irregular D(4)-quadruple cannot be extended to a quintuple,Vol.136, Acta Arith,2009,167-176.

[8]. Franusic Z., Diophantine quadruples in the ring Z[ ] Vol.9 , Math. Commun , 2004 , 141-148.

[9]. Franusic.Z, Diophantine quadruples in Quadratic Fields, Dissertation, University of Zagreb, 2005 (in Croatian)

[10]. Gopalan.M.A, Srividhya.G, Diophantine quadruples for Fibonacci and Lucas numbers with property D(4). Diophantus J.Math.2012; 1:15-18.
[11]. Gibbs.P, Diophantine quadruples and Cayley's hyperdeterminant, XXX Mathematics Archive math. NT/0107203
[12]. Gibbs.P, Adjugates of Diophantine Quadruples Integers . 2010;10:201-209.
[13]. Srividhya.G, Diophantine quadruples for Fibonacci numbers with property D(1). Indian journal of Mathematics and Mathematical Sciences.2009; 5:57-59.
[14]. Srividhya.G, Raghunathan.T.,Three Different Sequences of Diophantine Triples., JP Journal of Mathematical Sciences., Vol.20.,Iss.1 &2 ., .,2017 page no.1-18.