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Application of Kennelly'model of Running Performances to Elite Endurance Runners.

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Abstract: *The model of Kennelly between distance (D_{lim}) and exhaustion time (t_{lim}) has been applied to the individual performances of 19 elite endurance runners (World-record holders and Olympic winners) from P. Nurmi (1920-1924) to M. Farah (2012) whose individual best performances on several different distances are known. Kennelly's model ($D_{lim} = k\, t_{lim}{}^{\gamma}$) can describe the individual performances of elite runners with a high accuracy (errors lower than 2 %). There is a linear relationship between parameters k and exponents γ of the elite runners and the extreme values correspond to S. Coe (k = 15.8; γ = 0.851) and E. Zatopek (k = 6.57; γ = 0.984). Exponent γ can be considered as a dimensionless index of aerobic endurance which is close to 1 in the best endurance runners. If it is assumed than maximal aerobic speed can be maintained 7 min in elite endurance runners, exponent γ is equal to the normalized critical speed (critical speed/maximal aerobic speed) computed from exhaustion times equal to 3 and 12.5 min in these runners.*

I. Introduction

In 1906, Kennelly [1] studied the relationship between running distance (D_{lim}) and the time of the world records (t_{lim}) and proposed a power law of fatigue for the different types of exercise in humans and horses:

$D_{lim} = k\, t_{lim}{}^{\gamma}$ Equation 1

The value of exponent γ in power law is independent of scaling [2, 3]: the value of γ is independent of the expression of t_{lim} and D_{lim}. It is possible that γ is an expression of the endurance capability. Indeed, it is likely that the curvature of the t_{lim}-D_{lim} relationship depends on the decrease in the fraction of maximal aerobic metabolism that can be sustained during long lasting exercises. The t_{lim}-D_{lim} relationship is linear when γ is equal to 1. When D_{lim} is normalized to the value of D_{lim} at maximal aerobic speed (MAS), it can be demonstrated that the slope of the t_{lim}-D_{lim} curve is equal to γ (Fig. 1A) for t_{lim} equal to the exhaustion time at MAS (t_{MAS}).

At a running speed equal to MAS,

$D_{lim} = k\, t_{MAS}{}^{\gamma} = MAS\, t_{MAS}$ and therefore, $k = MAS * t_{MAS}{}^{1-\gamma}$ Equation 2

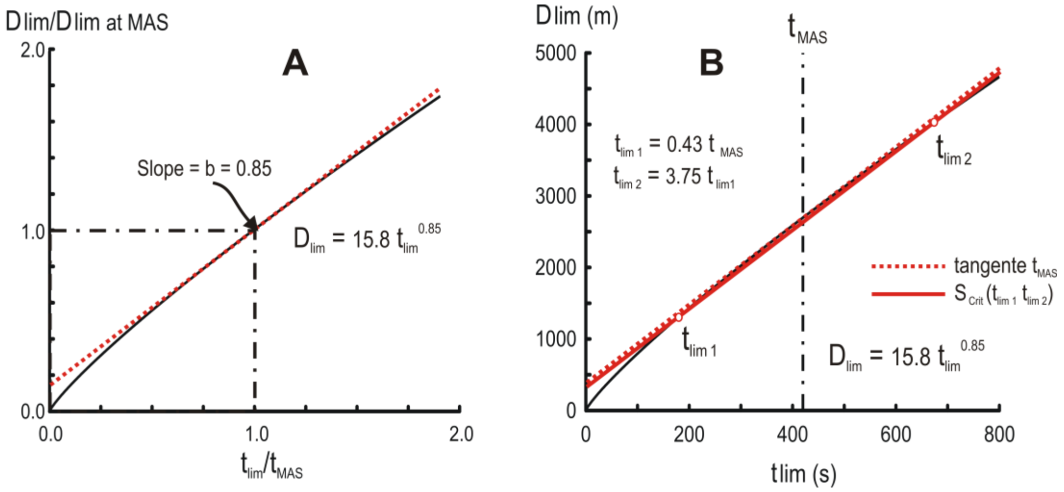

Figure 1: in A relationship between t_{lim} normalized to t_{MAS} and D_{lim} normalized to D_{lim} at MAS: the slope (red dotted line) of Kennelly curve (black curve) is equal to γ for t_{lim}/t_{MAS} = 1. In B, the value of the critical speed (red continuous line) computed from t_{lim1} and t_{lim2} is equal to the slope of Kennelly curve at t_{MAS} (red dotted line).

In 1954, Scherrer et al. [4] proposed a linear relationship between t_{lim} and the total amount of work performed at exhaustion (W_{lim}) for a local exercise (flexions or extensions of the elbow or the knee) performed at different constant power outputs (P) with t_{lim} rangeing between 3 and 30 minutes. This linear t_{lim}-W_{lim} relationship corresponded to a hyperbolic relationship between t_{lim} and P:

$W_{lim} = a + b\ t_{lim}$ and $t_{lim} = a/(P- b)$

In this model, parameter a was equivalent to a finite energy store whereas parameter b had the meaning of a power output which can be sustained during a long time and was called critical power. In 1958, Scherrer [5] proposed to apply the critical power concept to running or swimming exercises (critical speed) and to interpret the individual distance-time relationship as previously done for world records [6]. Thereafter, the concept of critical speed was applied to world records in different sports [7].

$D_{lim} = a + bt_{lim} = a + S_{Crit}\ t_{lim}$ Equation 3

The value of S_{Crit} is significantly correlated to the running speed at the 4 mmol blood lactate (8) and the lactate steady state running speed [9]. The S_{Crit}/MAS ratio is considered as an index of aerobic endurance, that is, the ability to sustain a high percentage of MAS during a long time. Therefore, exponent γ has been compared with S_{Crit}/MAS.

II. Methods

The individual power laws between D_{lim} and t_{lim} were determined by computing the regressions between the natural logarithms of D_{lim} and t_{lim}:

$\ln(D_{lim}) = \ln(k) + \gamma \ln(t_{lim})$ and $k = e^{\ln(k)}$

The short distances (< 1500 m) were not included in the present study. Indeed, it has been showed that the slopes of the regressions between velocity and the logarithm of time were different for races under and beyond 150-180 s [10]. This difference was the expression of the switch from the anaerobic metabolism that is needed for sprints to the aerobic metabolism used to supply energy for long distance races [10].

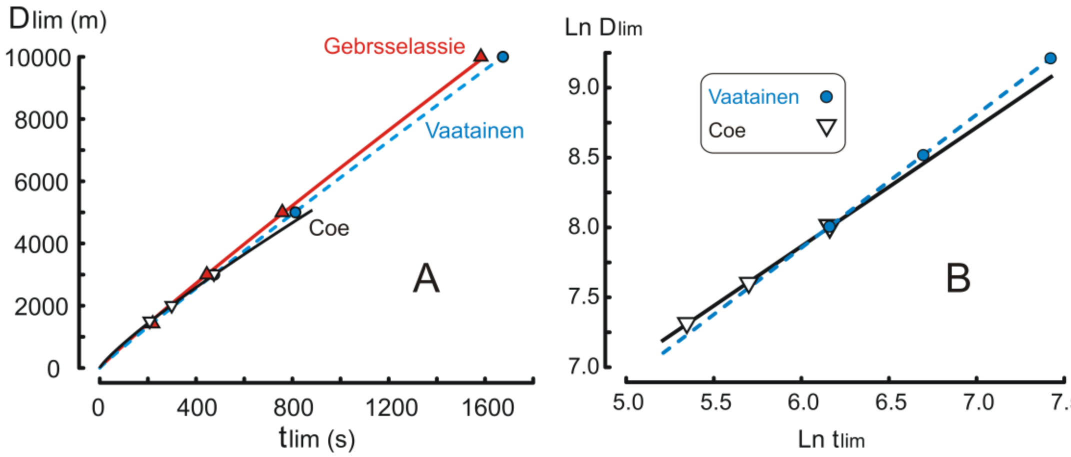

Figure 2: in A, three examples of the t_{lim}-D_{lim} relationships in elite runners. In B, relationships between the logarithms of t_{lim} and D_{lim} in two runners.

As the value of S_{Crit} depend on the range of t_{lim} [11], the individual values of S_{Crit} were estimated for the same ranges of t_{lim} by computing the values of D_{lim} corresponding to the same pairs of t_{lim} (t_{lim1} and t_{lim2}) from the individual power laws. The values of S_{Crit} were computed for different pairs of t_{lim} (3-12.5 min, 3-14 min and 3-16 min). Thereafter, it was assumed that MAS corresponds to the maximal velocity that can be sustained over 7 min (420 s) in elite endurance runners [12]. Therefore, the individual values of MAS in the present study were assumed to correspond to S_{420} and were computed from the individual D_{lim}-t_{lim} power laws:

$MAS = S_{420} = k\ (420)^{\gamma - 1}$

Finally, the values of S_{Crit} were normalized to MAS:

$S_{Crit}/MAS = S_{Crit}/S_{420}$

III. Results

The errors between the actual values of D_{lim} and their predicted values from Kennelly's model were lower than 2%. As an example, the data of Gebrsellassie are presented in Figure 3. The higher error (0.99 %) corresponded to the performance on 10,000 m. However, when the data of half-marathon and marathon were not included in the computation of the power law, the results of Gebrsellassie were slightly different: $\gamma = 0.95$ instead of 0.94 and k = 9.12 instead of 9.72.

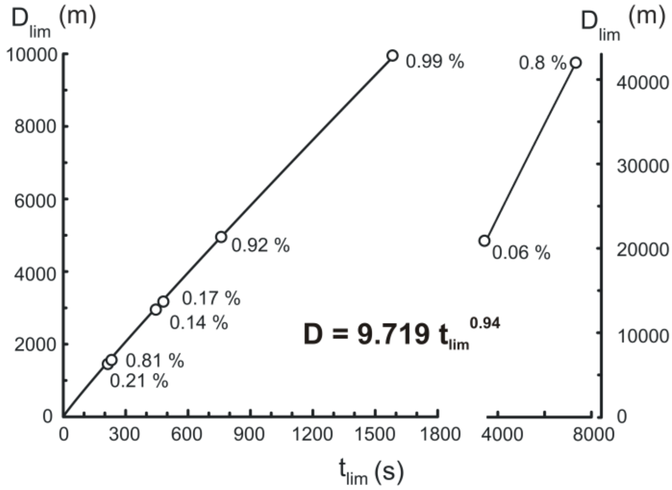

Figure 3: application of Kennely's model to the performances of Gebrselassie including half-marathon and marathon and percentages of errors on D_{lim}.

There is a linear relationship between parameters k and exponents γ of the elite runners (Fig.4 A). The extreme values correspond to S. Coe ($k = 15.8$; $\gamma = 0.851$) and E. Zatopek ($k = 6.57$; $\gamma = 0.984$).

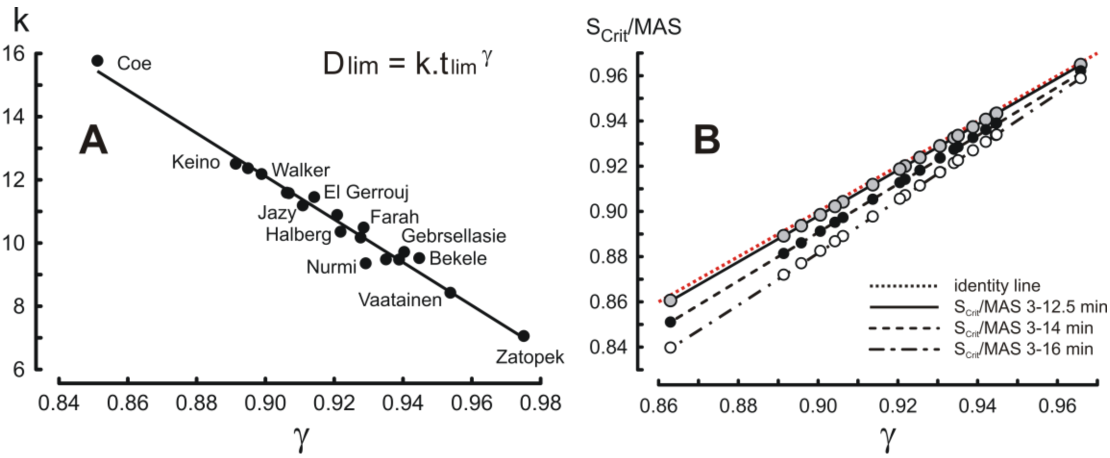

Figure 4: in A, relationship between the individual values of k and exponent γ in the 19 elite runners; in B, relationships between the individual values of exponent γ and S_{Crit}/MAS computed from different pairs of t_{lim} (3-12.5, 3-14 and 3-16 min).

The correlation coefficients of the linear regressions between γ and S_{Crit}/MAS were > 0.999 for the different ranges of t_{lim} in Fig. 4B (3-12.5, 3-14 and 3-16 min). The regression line was close to the identity line for t_{lim1} and t_{lim2} equal to 3 to 12.5 min, respectively.

IV. Discussion

The results of the present study indicate that Kennelly's model can describe individual performances of elite runners with a high accuracy because the errors in D_{lim} were lower than 2 %. The value of γ is close to 1 (0.984) for Zatopek and lower (0.851) for Coe who was a middle distance runner. In Fig. 2A, the t_{lim}-D_{lim} curve was almost linear for Gebrselassie in contrast with Coe. The hypothesis that exponent γ can be considered as a dimensionless index of aerobic endurance is confirmed by the linear relationship between γ and S_{Crit}/MAS that were almost equal when S_{Crit} was computed from two values of t_{lim} equal to 3 and 12.5 min.

The negative slope of the relationship between parameters k and γ (Fig. 4A) has two origins:

1. physiological origins because $k = MAS*t_{MAS}{}^{1-\gamma}$ (2) It is likely that the best runners on "short" distances (1500 m) have larger maximal anaerobic capacities and, consequently, higher values of t_{MAS}. On the other hand, the best performers in very long distance (20 km, marathon) have probably higher percentages of slow fibers and higher values of exponent γ. Consequently, these long distance endurance runners should have lower values of $t_{MAS}{}^{1-\gamma}$ and, therefore, lower values of k.

2. The effect of the underestimations of the performances in either the shortest or the longest distances on the linear $\ln(t_{lim})$-$\ln(D_{lim})$ relationship (Fig. 2B). An underestimation of the shortest distances induces a decrease in k and an increase in γ. Inversely, an underestimation of the longest distances induces an increase in k and a decrease in γ.

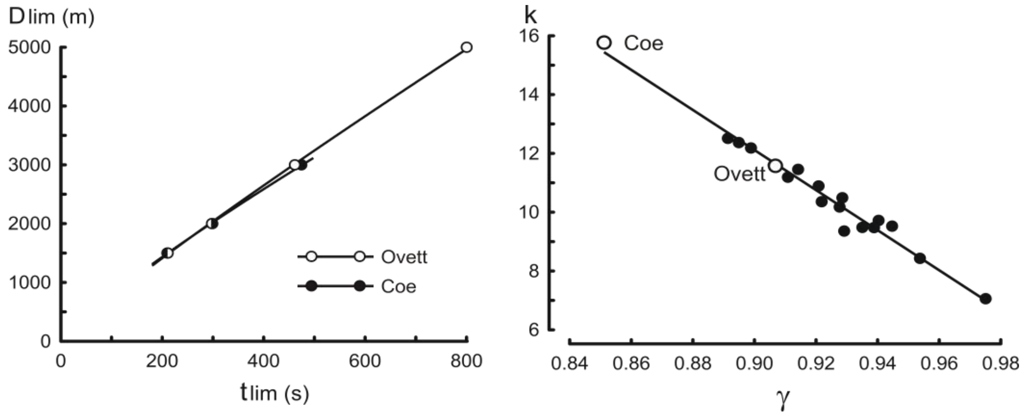

Figure 5: comparison of the running performances of Ovett and Coe (A) and their parameters k and γ (B).

The study of the performance of world elite runners is interesting. Indeed, the interpretation of the Kennelly's model assumes that the running data correspond to the maximal performance for each distance. The best performances of world elite runners generally correspond to the results of many competitions against other elite runners. The motivation is probably optimal during these races. However, it is assumed that the performances correspond to the same training and the same fitness level. Therefore, all the performances must be achieved within a few years. The differences in the results concerning Gebrselassie when half-marathon and marathon were not included suggest that this model is not perfect and cannot describe a very large range of distances. However, these differences could also be explained by the effects of age and ground (track vs road, slopes ...). The comparison of Ovett and Coe is also an illustration of the limits of the Kennelly's model. Indeed, the differences between Ovett and Coe for the performances in 1500 and 2000 m are around 1 second but the inclusion of longer distances (3000 m and 5000 m) induces large differences in the values of k and γ (Fig. 5). In fact, Ovett and Coe were the best runners on 800 and 1500 m but, as suggested [10], the performances in 800 m were not included in the model because they largely depend on anaerobic metabolism. The best performance for a given distance is maximal if the elite runner has run this distance many times, which was probably not the case for the distances equal to 3000 m and 5000 m, for Ovett and Coe. However, the difference between Ovett and Coe was not very high (0.907 *vs* 0.851) and corresponded to γ equal to 0.879 ± 0.028, i.e. ± 3.2%.

V. Conclusions

Kennelly's model can be used to describe the individual performances of elite runners with a high accuracy. There is a linear relationship between parameters k and exponents γ of the elite runners. Exponent γ can be considered as a dimensionless index of aerobic endurance which is close to 1 in the best endurance runners. In elite endurance runners, exponent γ is equal to the normalized critical speed (S_{Crit}/MAS) computed from exhaustion times equal to 3 and 12.5 min. However, further studies should verify 1) that Kennelly's model can accurately describe the individual running performance for a large range of distance; 2) that this model can be used in all the runners.

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