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# Thermohaline Convection With Variable Viscosity

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***Abstract:*** *The problem of double diffusive convection with variable viscosity confined between the two horizontal plates is investigated by the linear stability analysis. The transformed governing equations are numerically solved by using the Galerkin method. We have studied both stationary convection and oscillatory convection. The threshold values of Rayleigh number and wave number are computed and presented for various boundary conditions viz. rigid-rigid $(R/R)$, rigid-free $(R/F)$, free-rigid $(F/R)$ and free-free $(F/F)$ and for different values of physical parameters viz., salinity Rayleigh number $R_s$, Lewis number L, viscosity ratio c and Prandtl number $P_r$. For rigid-rigid boundary conditions we have studied the effect of $c, R_s$ on the vertical velocity and temperature eigenfunctions at the onset. It is observed that the salinity concentration stabilizes the dynamical system. The occurrence of co-dimension two bifurcation point $(CTP)$ is shown for various boundary conditions.*
***Keywords:*** *Thermohaline convection, Variable viscosity, Exponential fluid, Galerkin method.*

## I. Introduction

Natural convection in liquid-dominated hyper saline geothermal system is called thermohaline convection because it is driven by the thermal and solutal effects on liquid density. A problem of stability of two-component fluids subjected to uniform heating is of immense importance to geophysicists, hydrologists and soil scientists. A knowledge of the setting-up of convective currents in such a case is of much use in understanding the origin of mineral ores, which are either in the molten state (ores deep within the Earth at a very high temperature and may be in the molten state) or dissolved in fluids. In most of the situations a treatment of multi-component fluids is warranted since ores generally are found with other mineral impurities as well as mixtures of gases and / or liquids, e.g., crude oil associated with ground-water. For economical extraction of oil, one should have an idea of minimum perturbations needed to initiate convection in such fluids and this in turn necessitates the study of multi-component fluids saturating in the Earth's crust. In most of the cases, due to high-temperature variations involved [of the order of 250 $K$[Horne and O'Sullivan [1]], viscosity cannot be taken as a constant. However, most of the works in this field have been carried out by assuming viscosity to be a constant. For density variations, the assumption of a Boussinesq fluid has been made, which means that the changes in density are significant only in the body force term, while this density may be treated as a constant everywhere else, in the equation of motion [see Chandrasekhar [2] and Joseph [3] for a detailed discussion].

Setting-up of convective currents in two-component fluids is a different process from that of single-component fluids. This is so because in a single-component fluid, a force due to the density gradient is caused by the variation of density, which is due to the variation in temperature whereas in multi-component fluids a force due to density gradient is caused by the variation of density which is due to the variations in both concentration and temperature, and in turn lead to setting up of instability earlier.

In the case of a single-component fluid when the density of the fluid decreases vertically upwards it is necessary for a stable configuration, whereas in a multi-component fluid this is not necessarily so, and each component can diffuse with respect to each other. As the diffusivity of mass is less than the diffusivity of heat, a displaced portion of the fluid loses its heat content faster than its salt content. The resulting forces due to a density gradient may tend to increase the displacement from the original position causing instability. In some cases, it may cause over stable motion also. This is understood by considering the layer of a fluid whose density decreases with the increase of temperature and increases with the concentration of salt ([4]-[6]). Suppose that when both the concentration and temperature decrease vertically upwards, then the warmer liquid tend to rise to a cooler region. Since the concentration diffuses slowly ($K_s < K$, i.e., the diffusivity of heat $K$ is usually much greater than the diffusivity of a salt), and also heavier, the force due to the density gradient tend to bring it down. As this concentration has spent some time on the cooler region, it is heavier than the ambient fluid and hence it overshoots in its initial position, thus causing oscillations of increasing amplitude.

Earlier workers (e.g.,Veronis [4], Nield [7] and Shirtcliffe [8]) studied the nature of thermohaline convection in the esence of a stable salinity gradient. Turner [9] studied in detail both experimentally and theoretically the nature of the thickness of a growing layer at the bottom. Brains and Gill [10] theoretically obtained the relation for the critical salinity Rayleigh number. In all these above works viscosity is considered as a constant.

In this paper an attempt is made to study the effect of variable viscosity on the convective instability of a two-component fluid. Various cases of stabilizing and destabilizing gradients of concentration and temperature are analysed. The linear stability analysis of this physical model is analysed by using the Galerkin method. We have considered the Galerkin expansion for all the dependent variables in terms of the orthogonal functions satisfying different combinations of the boundary conditions namely, $(R/R), (R/F), (F/R)$, and $(F/F)$. We present the results of stationary and oscillatory instabilities for different values of salinity Rayleigh number *Rs*, viscosity ratio *c* and Prandtl number *Pr* for the temperature dependent variable viscosity fluids, namely, the exponential fluids. It should be noted that the term oscillatory convection refers to the small amplitude convection at the onset of convection. Oscillatory fluid motions can occur in double diffusive convection where the density contribution from the temperature component is destabilizing and from the solute component is stabilizing. These exponential fluids were first used by Torrence and Turcotte [11]. It is shown that the numerical results obtained in the present study are in good agreement with the results of Stengel et. al. [12] for the onset of stationary convection in the absence of salinity with variable viscosity. Also for constant viscosity, with or without salinity our results coincide with those of Chandrasekhar [2] (Table 1 and 2).

The scheme of the paper is the following one. In Section 2 we obtain the non-dimensional perturbation equations of the thermohaline convection for exponential fluids. In Section 3 we have studied the linear stability analysis. Numerical results are presented in Section 4. Finally, conclusions are made in Section 5.

## II. Mathematical Formulation

A quiescent layer of a two-component Boussinesq fluid, is assumed to be confined between the two boundaries at $z = -d/2$and $= d/2$.. The viscosity of the fluid is assumed to be a function of temperature. The governing equations for thermohaline convection with variable viscosity in dimensional form are given by:
the equation of continuity

$$\nabla'.\vec{V}' = 0, \tag{2.1}$$

the momentum equation

$$\rho_0\left(\frac{\partial}{\partial t}\nabla'.\vec{V}'\right)\vec{V}' = -\nabla' P' + \rho_0\left(K_1\hat{\imath} + K_2\hat{\jmath} + K_3\hat{k}\right) + \rho\vec{g}, \tag{2.2}$$

the heat equation

$$\frac{\partial T'}{\partial t'} + \left(\nabla'.\vec{V}'\right)T' = K\nabla'^2 T', \tag{2.3}$$

and the concentration equation

$$\frac{\partial S'}{\partial t'} + \left(\vec{V}'.\nabla'\right)S' = K_s\nabla'^2 T', \tag{2.4}$$

where in eq. (2.2)

$$K_1 = 2\mu\frac{\partial^2 \partial u'}{\partial x'^2} + \frac{\partial}{\partial y'}\left[\mu\left(\frac{\partial u'}{\partial y'} + \frac{\partial u'}{\partial x'}\right)\right] + \frac{\partial}{\partial z'}\left[\mu\left(\frac{\partial w'}{\partial x'} + \frac{\partial u'}{\partial z'}\right)\right],$$

$$K_2 = 2\mu\frac{\partial^2 \partial v'}{\partial y'^2} + \frac{\partial}{\partial z'}\left[\mu\left(\frac{\partial v'}{\partial z'} + \frac{\partial w'}{\partial y'}\right)\right] + \frac{\partial}{\partial x'}\left[\mu\left(\frac{\partial u'}{\partial y'} + \frac{\partial v'}{\partial x'}\right)\right],$$

$$K_3 = \frac{\partial}{\partial z'}\left[2\mu\frac{\partial w'}{\partial z'}\right] + \frac{\partial}{\partial x'}\left[\mu\left(\frac{\partial w'}{\partial x'} + \frac{\partial u'}{\partial z'}\right)\right] + \frac{\partial}{\partial y'}\left[\mu\left(\frac{\partial u'}{\partial z'} + \frac{\partial u'}{\partial z'}\right)\right].$$

The dependent variables are, the velocity $\vec{V}'(u', v', w')$, density $\rho$, pressure $P'$,temperature $T'$, solute mass concentration $S'$ and viscosity$\mu$. The remaining quantities are assumed to be constant. In the medium the diffusivities of heat and concentration are denoted by $k$ and $k_s$,respectively.
When a salt is added to a volume of a water the volume increases, and that tends to decrease the density of the salt, but the total density of the fluid increases. Let $\beta_s$ measures this density increase. Thus in the thermohaline convection density variations depend on both diffusive mechanisms. Thusthe variation may be taken as ([4], [5] and [13]):

$$\rho = \rho_0[1 - \alpha(T' - T_1') - (S' - S_b')], \tag{2.5}$$

$$\rho = \rho_0[1 - \alpha(T' - T_1') + \beta_s(S' - S_b')], \tag{2.6}$$

with $\alpha = -\frac{1\partial}{\rho_0}\left(\frac{\partial\rho}{\partial T'}\right)$and$\beta_s = \frac{1\partial}{\rho_0}\left(\frac{\partial\rho}{\partial S'}\right)$.

Here $\rho_0$ is the mean density of the system, $T^{'}$ and $S^{'}$ are temperature and salinity concentration of the system, a is thermal expansion coefficient and $\beta_s$ is the coefficient of density and it increases with respect to salinity and $S_b^{'}$ is the salinity concentration at the bottom boundary. The choice of the density variation will depend upon the physics of the two-component system. If the heavier component of the fluid is introduced into the system, then due to diffusion of mass, there results an increase in the density of the mixture in which case the relation (2.5) is appropriate. If the lighter component is introduced then mass diffusion lowers the density of the mixture, hence the validity of relation (2.6).

The following four cases are considered (for the density variation given by eq. (2.5)):

(a) heated from above (stabilizing) and salted from above (destabilizing);
(b) heated from below (destabilizing) and salted from below (stabilizing);
(c) heated from below (destabilizing) and salted from above (destabilizing);
(d) heated from above (stabilizing) and salted from below (stabilizing).

In this paper the case (b) is discussed in detail. It can be noted that, when salting from below the concentration decreases as the depth of the layer increases upwards.

The conduction state is characterized by

$$\vec{V}^{'} = 0, T_s^{'} = T_1^{'} - (\Delta T^{'}/d)z^{'}, S_s^{'} = S_b^{'} - (\Delta S^{'}/d)z^{'}, P_s^{'} = P_0^{'} + z^{'}g + \alpha\beta g\frac{z^{'2}}{2}.$$

In the conductive state solutions suffix s stands for static. The uniform temperature gradient $\Delta T^{'}$ and concentration

Gradient $\Delta S^{'}$ are positive if the corresponding quantities decrease upwards. The static solutions, $T_s^{'}$ of $T^{'}$ gives destabilizing effect and $S_s^{'}$ of $S^{'}$ gives stabilizing effect.Now the velocity, temperature, pressure and concentration perturbations written as

$$\vec{V}^{'} = \vec{V}^{'}s + \vec{V}^{'*}, T^{'} = T_s^{'} + \theta^{'*},\ P^{'} = P_s^{'} + P^{'*} \text{and} C^{'} = S^{'} + S_s^{'*},$$

respectively.The scaling $k/d, \beta d, d^2/k$ , $\rho_0 k^2 d^{-2}$ and $\beta_s d$ are used to non-dimensionalize the velocity, temperature, time, pressure and concentration, respectively. Here $\beta_s d = \Delta C^{'}$ and $\beta d = \Delta T^{'}$. Thus, the non-dimensional governing equations in two dimensions viz., (*x*, *z*)planes are given by:

$$\nabla^{'}.\vec{V} = 0, \qquad (2.7)$$

$$\frac{1}{P_r}\left[\frac{\partial \vec{V}}{\partial t} + (\nabla^{'}.\vec{V})\vec{V}\right] = -\frac{\nabla P}{P_r} + (R_T\theta - R_s C)\hat{e}_z + \hat{e}_z\left[2f\frac{\partial^2 u}{\partial_x{}^2} + \frac{\partial}{\partial z}\left(f\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)\right)\right] + \hat{e}_y\left[\frac{\partial}{\partial z}\left(f\frac{\partial v}{\partial z}\right) + \frac{\partial}{\partial x}\left(f\frac{\partial v}{\partial x}\right)\right] + \hat{e}_z\left[\frac{\partial}{\partial z}\left(f\frac{\partial w}{\partial z}\right) + \frac{\partial}{\partial x}\left(f\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)\right], \qquad (2.8)$$

$$\frac{\partial\theta}{\partial t} + (\vec{V}.\nabla)\theta = w + \nabla^2\theta, \qquad (2.9)$$

$$\frac{1}{L}\left[\frac{\partial C}{\partial t} + (\vec{V}.\nabla)C\right] = \frac{w}{L} + \nabla^2 C, \qquad (2.10)$$

where $\hat{e}_x$, $\hat{e}_y$ and $\hat{e}_z$ are the unit vectors along *x*, *y* and *z*– directions respectively. For our convenience we have omitted the symbol '*' from eqn. (2.7)-(2.10). The *z*-component of the curl of curl of the momentum equation (2.8) is

$$\left(\frac{1}{P_r}\frac{\partial}{\partial t} - f\nabla^2\right)\nabla^2 w - 2\frac{df}{\partial \text{z}}\nabla\frac{\partial w}{\partial \text{z}} + \frac{d^2 f}{\nabla \text{z}^2}\left(\frac{\partial^2}{dx^2} + \frac{\partial^2}{dz^2}\right)w + R_T\nabla_h^2\theta - R_s\nabla_h^2 C = \hat{e}_z.\frac{1}{P_r}\left[\nabla\times\nabla\times\nabla(\vec{V}.\nabla)\vec{V}\right]. \qquad (2.11)$$

The variable viscosity, $v(z)(=\mu/\rho_0)$ is given by $v(z)(= v_0 f(z)$, where $v_0$ is the reference value of the kinematic viscosity evaluated at $T_0$ and $f(= v/v_0)$ is the dimensionless viscosity.

Following Stengel et al.[12], the exponential dependence of the viscosity is assumed to be:

$$f = \exp\left[c(T_0 - T)\right], \qquad (2.12)$$

where

$$c = \log\left(\frac{v_{max}}{v_{min}}\right) = \log\left(\frac{1+\gamma}{1-\gamma}\right), \qquad 0 \le \gamma < 1.$$

The temperature dependence (2.12) appears important both in industrial applications and in geophysics [11].

## III. Linear Stability Analysis

In this section we study the effect of physical parameters on the onset of convection. The normal mode expansion of the dependent variables is assumed to be of the form

$[w, \theta, C] = [W(z),\ \Theta(z),\ C(z)]\exp\,(iqx + pt),$ (2.13)

where $W(z),\ \Theta(z)\ and\ C(z)$ are the eigenfunctions, $p$ denotes the growth rate of the perturbations and $q$ denotes the horizontal wavenumber. Using the
above set of solutions (3.13), into the linear parts of the eqs.(2.8)-(2.10), we get

$\frac{p}{P_r}(D^2 - q^2)W = (D^2 f)(D^2 + q^2)W + (2Df)(D^2 - q^2)^2 + DW + f(D^2 - q^2)^2 W - R_T q^2 \Theta R_s q^2 C,$(3.14)

$p\Theta = (D^2 - q^2)\Theta + W,$ (3.15)

$\frac{p}{L}C = (D^2 - q^2)C + \frac{W}{L}.$ (3.16)

Here $D = d/dz$ is the differential operator, $W(z)$ is the *z*-part of the vertical component of the velocity, $\Theta(z)$ is the *z*-part of temperature, *C* is the *z*-part of concentration and $L = k_s/k$, (where $k_s$ and *k* are the diffusivities of the slower diffusing and faster diffusing components respectively) is the Lewis number.

### 3.1. Boundary Conditions

The physical model given in eqs. (3.14)-(3.16) are analyzed by considering the following four types of boundary conditions namely, $(R/R)$, $(R/F)$, $(F/R)$, and $(F/F)$.

(i) No slip condition on the top and bottom boundaries $(R/R)$.i.e.,
$W = DW = \Theta + C = 0 \text{at} z = 1/2, z = -1/2.$
(3.17)

(ii) Stress-free condition at the top $(z = -1/2)$ and no slip condition at the bottom $(z = -1/2)$ boundaries $(F/R)$.i.e.,

$W = D^2W = \Theta = C = 0 \text{at} z = 1/2,$
$W = DW = \Theta = C = 0 \text{at} z = -1/2,$ (3.18)

(iii) No slip condition at the top $z = 1/2$ and stress free condition at the bottom $(z = -1/2)$ boundaries $(R/F)$.i.e.,

$W = DW = \Theta = C = 0 \text{at} z = 1/2,$
$W = D^2W = \Theta = C = 0 \text{at} z = -1/2,$ (3.19)

(iv) Stress-free condition at the top and bottom boundaries $(F/F)$. i.e.,

$W = D^2W = \Theta = C = 0 \text{at} z = 1/2, z = -1/2$ (3.20)

In the above equations (3.17)-(3.20), *D* denotes differentiation with respect to the vertical coordinate *z*.

The unknown variables $W(z)$, $C(z)$ and $\Theta(z)$ are expanded in terms of the following complete sets of trial functions, that satisfy the homogeneous boundary conditions(3.17)-(3.20)

$W(z) = \sum_{n=1}^{\infty} a_n W_n(z), \quad C(z) = \sum_{n=1}^{\infty} b_n C_n(z) \text{and}\ \Theta(z) = \sum_{n=1}^{\infty} c_n \Theta_n(z),$ (3.21)

where $W_n(z)$, $C_n(z)$ and $\Theta_n(z)$ are the trial functions that satisfy the homogeneous boundary conditions (3.17)-(3.20), and the coefficients $a_n$, $b_n$ and $c_n$ are unknown constants. The trial functions that satisfy the no-slip boundary conditions (3.17) are chosen as:

$$W_n(z) = \begin{cases} \frac{\cosh(\alpha_n z)}{\cosh\left(\frac{\alpha_n}{2}\right)} - \frac{\cos(\alpha_n z)}{\cos\left(\frac{\alpha_n}{2}\right)}, & n \text{ is odd} \\ \frac{\sinh(\alpha_n z)}{\sinh(\alpha_n/2)} - \frac{\sin(\alpha_n z)}{\sin(\alpha_n/2)}, & n \text{ is even} \end{cases} \quad (3.22)$$

where the constants $\alpha_n$ are zeros of:

$$\tanh(\alpha_n/2) + \tan(\alpha_n/2) = 0, \quad n \text{ is odd}$$
$$\coth(\alpha_n/2) + \cot(\alpha_n/2) = 0, \quad n \text{ is even}$$

The function $C_n(z)$ is given by

$$C_n(z) = \begin{cases} \cos(n\pi z), & n \text{ is odd} \\ \sin(n\pi z), & n \text{ is even} \end{cases} \qquad (3.23)$$

The function $\Theta_n(z)$ is given by

$$\Theta_n(z) = \begin{cases} \cos(n\pi z), & n \text{ is odd} \\ \sin(n\pi z), & n \text{ is even} \end{cases} \qquad (3.24)$$

Similarly, we can choose the trial functions for the remaining boundary conditions viz., *R/F*, *F/R* and *F/F*. According to the Galerkin method, the residuals are required to be orthogonal to the trial functions. This method is used to obtain an infinite set of linear homogenous algebraic equations for the unknown coefficients, namely,$a_n$, $b_n$and $c_n$in eq.(3.21). In order to obtain a finite number of linear homogeneous algebraic equations, we truncate expansions of $W_n(z)$, $C_n(z)$and $\Theta_n(z)$ in eq. (3.21) at *M*, *N* and *K* terms respectively. Thus we are left with matrix eigenvalue problem as

$$(U - pV)X = 0, \qquad (3.25)$$

where$U$, $V$ and $X$ are the matrices and they are given by

$$U = \begin{pmatrix} B_{ji} & C_{ji} & D_{ji} \\ F_{ji} & G_{ji} & H_{ji} \\ J_{ji} & K_{ji} & L_{ji} \end{pmatrix}, V = \begin{pmatrix} A_{ji} & 0 & 0 \\ 0 & E_{ji} & 0 \\ 0 & 0 & I_{ji} \end{pmatrix}, \ X = \begin{pmatrix} a_n \\ b_n \\ c_n \end{pmatrix}. \qquad (3.26)$$

The matrix elements in eq.(3.26) are defined as

$$A_{ji} = \frac{1}{P_r}\langle W_j D^2 W_i - q^2 W_j W_i \rangle,$$
$$B_{ji} = \langle (D^2 f)\left(W_j D^2 W_i + q^2 W_j W_i\right) + (2Df)\left(W_j D^3 W_i + q^2 W_j W_i\right)$$
$$+ f\left(W_j D^4 W_i - 2q^2 W_j D^2 W_i + q^4 W_j W_i\right)\rangle$$
$$C_{ji} = -R_T q^2 \langle W_j \Theta_i \rangle,\ D_{ji} = -R_s q^2 \langle W_j C_i \rangle, E_{ji} = \langle \Theta_j \Theta_i \rangle,$$
$$F_{ji} = \langle \Theta_j W_i \rangle,\ D_{ji} = \langle \Theta_j D^2 \Theta_i - q^2 \Theta_j \Theta_i \rangle,$$
$$H_{ji} = \langle 0 \rangle,\ I_{ji} = \frac{1}{L}\langle C_j C_i \rangle,\ \ J_{ji} = \frac{1}{L}\langle C_j W_i \rangle,$$
$$K_{ji} = \langle 0 \rangle,\ L_{ji} = \langle C_j D^2 C_i - q^2 C_j C_i \rangle \qquad (3.27)$$

and the angular bracket expression represents

$$\langle h_1(z) h_2(z) \rangle = \int_{-1/2}^{1/2} h_1(z) h_2(z)\, dz,$$

where$h_1(z)$ and $h_2(z)$are the orthogonal functions. For the non-trivial solutions, the system of eqs. (3.25) gives the characteristic equation as:

$$|U - pV| = 0. \qquad (3.28)$$

The system of eqs. (3.25) is solved as a generalized eigenvalue problem. This task is accomplished numerically, by making this infinite set of equations to finite by the numerical truncation. The real parts of the eigenvalues, say, $Re(p_k) < 0$ determine the stability of the system. If $Re(p_k) < 0$ for all *k*, the system is said to be stable. If $Re(p_k) > 0$ for at least one value of *k*, the system is unstable. The marginal stability curve corresponds to the case when one of the eigenvalues satisfies $Re(p_k) = 0$. The eigenvalue $p$ is a function of the physical parameters $R_T$, $q$, $R_s c$, $P_r$, $L$and $f(z)$.The parameter $R_T$ determines the onset of instabilities and depends on the physical parameters$R_T$, $q$, $R_s c$,$P_r$, $L$ and$f(z)$.Only those $R_T$ values that are real, positive and finite are considered to be physically meaningful. For any $q$ we take the smallest of these meaningful $R_T$. We obtain critical value of $R_T$ by iterating $q$ until the marginal stable value of $R_T$ is minimized. The convective system can be unstable to either stationary convection or oscillatory convection at the onset of instability. The occurrence of stationary convection or oscillatory convection in the convective system depends on the physical parameters.

### 3.2. Stationary Convection

The stationary convection is obtained when one of the eigenvalues, $p_k$ vanishes. Thus, the Rayleigh number for the stationary convection($R_T$= $R_T$) is Table 1. Comparison of critical Rayleigh numbers of convection between the present results and those of Chandrasekhar [2] for the constant viscosity ($c$ = 0).

| | Top and bottom boundaries | $R/R$ | $R/F$ | $F/R$ | $F/F$ |
|---|---|---|---|---|---|
| $R_s = 0$ Stationery convention | Present results | 1707.7758 | 1100.4712 | 1100.4712 | 657.01535 |
| | Chandrasekhar [2] | 1707.7620 | 1100.650 | 1100.650 | 657.5110 |

**Table 2.** Comparison of critical Rayleigh numbers of stationary convection between the present results and those of Stengel et al. [12], for the viscosity ratio ($c$ = 6) in the absence of salinityconcentration ($R_s$).

| Top and bottom boundaries | $R/R$ | $R/F$ | $F/R$ | $F/F$ |
|---|---|---|---|---|
| Present results | 2167.0125 | 1678.3256 | 1630.3212 | 1258.0123 |
| Stengel et al. [2] | 2167.94 | 1678.330 | 1629.510 | 1258.510 |

obtained from eq. (3.28) for$i = j = 1$ as:

$$R_s = \frac{I_{11}D_{11}G_{11} + B_{11}H_{11}J_{11} + E_{11}D_{11}J_{11} - B_{11}G_{11}L_{11}}{C(I_{11}H_{11} - E_{11}L_{11})} \quad (3.29)$$

It can be observed that the matrix elements on the right hand side of the above eq. (3.29) are independent of $P_r$. Hence the critical Rayleigh number for stationary convection is independent of $P_r$. The critical Rayleigh number$R_T = R_{sc}$and the critical wave number $q = q_{sc}$ for the onset of stationary convection depends on the physical parameters $R_s$, $L$ and $c$.

### 3.3. Oscillatory Convection

The condition for oscillatory convection are given by$p_k = \pm iw$, where $w$ is the frequency of the oscillations and $w^2 > 0$,where

$$\omega^2 = \frac{1}{A_{11}F_{11}K_{11}}\left[B_{11}F_{11}L_{11-} + E_{11}C_{11}K_{11}R_T - I_{11}D_{11}F_{11} + A_{11}G_{11}L_{11} + B_{11}G_{11}K_{11} - A_{11}H_{11}J_{11}\right] (3.30)$$

The Rayleigh number for the oscillatory convection ($R_T = R_0$)is given by:

$$R_0 = \frac{M}{K_{11}C_{11}(I_{11}H_{11}A_{11}F_{11} + E_{11}B_{11}F_{11}K_{11} + E_{11}A_{11}G_{11}K_{11})}, \quad (3.29)$$

where

$$\begin{aligned} M = {} & 2B_{11}G_{11}L_{11}A_{11}F_{11}K_{11} + B_{11}^2F_{11}K_{11}^2G_{11} + B_{11}^2F_{11}^2K_{11}L_{11} - B_{11}K_{11}I_{11}D_{11} \\ & + A_{11}G_{11}^2K_{11}^2B_{11} - A_{11}^2G_{11}K_{11}H_{11}J_{11}A_{11}F_{11}^2L_{11}^2B_{11} - A_{11}F_{11}^2L_{11}I_{11}D_{11} \\ & + A_{11}^2F_{11}L_{11}^2G_{11} - A_{11}^2F_{11}L_{11}H_{11}J_{11} - E_{11}D_{11}J_{11}A_{11}F_{11}K_{11} \end{aligned}$$

The quantities in eqs. (3.29), (3.30) and (3.31) are obtained from eq. (3.27) for $i = j = 1$. The critical Rayleigh number at$R_T = R_{oc}$and critical wave number at$q = q_{oc}$, for various types of boundary conditions, are computed when$w^2 > 0$. The threshold values of$R_{oc}$ and $q_{0c}$ depends on $L$, $P_r$, $R_s$ and $c$. For the present physical system, we have observed that overstability can occur when the $P_r$ is finite. The time dependent convective flow exists when $R = R_{oc}$. These numerical values of Rayleigh number and wavenumber are obtained by using Maple software. At$R_T = R_{sc}$, $q = q_{sc}$and at $R_T = R_{oc}$, $q = q_{oc}$ we get the pitchfork and the Hopf bifurcations, respectively. The pitchfork and the Hopf bifurcation are known as the primary bifurcations. Pitchfork bifurcation arises when the characteristic eq. (3.28) possesses a simple zero eigenvalue. Hopf bifurcation arises when a pair of purely imaginary complex conjugate eigenvalues is obtained from the characteristic eq. (3.28). The secondary bifurcations, viz., Tokens-Bogdanov bifurcation point($w^2 = 0$)and co-dimension two bifurcation point ($CTP$) ($w^2 > 0$), occurs on combining Rayleigh numbers and wavenumbers of stationary convection and oscillatory convection. At the ($CTP$), we get$R_{sc} = R_{oc}$but $q_{sc} \neq q_{oc}$.The intersection point of the neutral curves of stationary convection and oscillatory convection in the $(c, R_c)$-plane gives ($CTP$) and is discussed in detail in Section 4.4.

## IV. Numerical Results And Discussions

The parameters which are influencing the criterion for the onset of convection are the salinity Rayleigh number $R_s$, Lewis number $L$, Prandtl number$P_r$ and the viscosity ratio $c$. The salient characteristics of these parameters are exhibited in Figures (1) to (12). In all these figures green lines stand for stationary convection and red lines stand for oscillatory convection.

In Figure 1, we have shown the effect of $c$ and$R_s$ on critical Rayleigh number for large$P_r$.Figures l(a-d) are plotted for$R/R$, $R/F$ ,$F/R$ and $F/F$ boundary conditions, respectively. For large $P_r$ we get only the stationary convection at the onset. Figure l(a) shows that as $c$, increases initially $R_{rsc}$ increases and approaches to a maximum value, then starts decreasing for further increment values of $c$. Let $c = c^*$ denotes the maximum value on a curve. For $c < c^*$, as $c$ increases the onset of convection increases and for $c > c^*$, as $c$ increases the onset of stationary convection decreases. This figure also shows the effect of $R_s$ on the onset. As $R_s > 0$, increases the onset of convection also increases. This implies that the salinity concentration inhibits the onset of convection. Similar results have been observed for the remaining boundary conditions. Due to the density of concentration fluids, the critical Rayleigh numbers of thermohaline convection are larger than the critical Rayleigh number of pure fluids.

In Figures 2(a-d), we have shown the effect of $R_s$ and $c$ on $q_{sc}$ for$R/R$, $R/F$, $F/R$ and $F/F$ boundary conditions, respectively. Figure 2(a), shows as $c$ increases $q_{sc}$ initially decreases and reaches to a minimum value. Further increment in the values of $c, q_{sc}$increases monotonically.As $R_s$increases $q_{sc}$.Where as for other boundary conditions $q_{sc}$ decreases considerably as $c$ increases and after attaining a minimum value $q_{sc}$ increases monotonically for further increment values of $c$.

In Figures 3(a-d), we have shown the effect of $L$on $R_{sc}$. From these figures we can observe that as $L$ increases the value of $R_{sc}$ deceases, this implies that the effect of $L$ destabilizes the convective system. Figures 4(a-d), show that the effect of $L$ on $q_{sc}$. For $R/R$ boundary conditions $q_{sc}$ initially decreasesand reaches to a minimum value slowly as $c$ increases and increases monotonically as $c$ increases further. For $R/F$ and $F/R$ boundary conditions, $q_{sc}$ increases slowly as $c$ increases and when $c$ is small, for higher values of $c$ as $c$ increases, $q_{sc}$ increases monotonically. Figure 4(d), shows that as $c$ increases, initially $q_{sc}$ decreases and increases rapidly for further incremental values of $c$. This behavior of $q_{sc}$ changes when$c \approx 10$. In Figures 5 to 7, we have shown the occurrence of oscillatory convection for $R/R$ and $F/F$ boundary conditions. In these figures we have studied the effect of$R_s$,$L$ and $P_r$ on the onset of convection. The behavior of $R_{oc}$depends on $c$ and is similar to $R_{sc}$ with respect to $c$. It is observed that $R_{0c}$ increases either $R_s$or $P_r$ increases, but decreases as $L$ increases. These figures show that for finite $P_r$, $R_{oc}$is less than $R_{sc}$, i.e., the convective system is unstable to oscillatory convection. From these figures we can also observe that either $R_s$ or $P_r$ increases, $q_{oc}$ decreases, whereas $q_{oc}$ decreases as $L$ increases. The dependence of $q_{oc}$ is similar to that of $q_{sc}$with increasing values of $c$. Figures 8 (a-c), show the effect of viscosity ratio, $c$, for physical parameters $R_s$, $L$, $P_r$ on w1. Figure 9, shows the vertical velocity eigenfunction for $W(z)$ as a function of the depth $z$, $L$ and $R_s$ with $R/R$boundary conditions for several values of viscosity ratio $c$. From this figure, we can observe that for $c = 10$ the velocity is symmetric about the midpoint of the layer. As the ratio of viscosity to the top and bottom boundaries increases, velocity of the fluid particles where it takes the maximum value is located near the bottom of the fluid layer where the fluid is less viscous. Figure 10, shows the variation of temperature eigenfunction $0(z)$ as a function of depth, $c$ and $R_s$ with $R/R$boundary conditions and for several values of the viscosity ratio $c$. From this figure we can observe that for small values of $c, \Theta(z)$ is symmetric about the mid point of the fluid layer. As $c$ increases the temperature perturbation becomes confined to small region near the bottom of the fluid layer where the fluid is less viscous. Figure 11, shows the effect of $R_s$ for different values of viscosity variation c. The straight lines which represent the variation of the critical Rayleigh number with the salinity Rayleigh number are lines with positive slopes. The temperature dependent viscosity variation (i.e., the values for $c \neq 10$) displaces the straight line upwards. The increase in the critical Rayleigh numbers clearly show that the system stabilizes with increasing viscosity ratio c and salinity gradient $(R_s)$.

Figure 12, shows the occurrence of stationary convection and oscillatory convection for the given values of physical parameters. Figure 12, is plotted for fixed values of $c = 0$and $c = 5$. In this figure green line correspond to stationary convection and red line correspond to oscillatory convection. Green line is plotted for $R_{sc}$ and red line is plotted for $R_{0c}$ for different values of $R_s$. From this figure we can observe that, we get only stationary convection at the onset. The values on the green and red lines represent the occurrence of the pitchfork and Hopf bifurcations, respectively. These two bifurcations are known as primary bifurcations (co-dimension one bifurcations). The behavior of $W(z)$ and $\Theta(z)$ of oscillatory convection near the onset is similar to the behavior of $W(z)$and $\Theta(z)$of stationary convection. Due to this reason we have omitted the numerical results related to $W(z)$and $\Theta(z)$of oscillatory convection.

## V. Conclusions

The exponential behavior of the fluid is considered between two horizontal boundaries in which the lower boundary is heated from below and also salted from below. The thermohaline convection with temperature dependent variable viscosity has been studied by using linear stability analysis for $R/R$, $R/F$, $F/R$ and $F/F$ boundary conditions. Since the system is double diffusive, at the onset we can get either stationary convection or oscillatory convection.

The occurrence of the type of convection is analyzed by using Prandtl number. For large Prandtl number, we have obtained only stationary convection at the onset. Based on the numerical results the parameter range of viscosity ratio region can be divided as; low viscosity ratio region moderate viscosity ratio region and large viscosity ratio region. For finite $P_r$ the results show that the system is unstable always to oscillatory convection.

- In the low viscosity ratio region as c increases $R_{sc}$ is nearly a constant.In the moderate viscosity region as c increases $R_{sc}$ increases and attainsa maximum value at $c = c^*$. In the large viscosity ratio region, as $c$increases $R_{sc}$ decreases. The decreasing value of $R_{sc}$ indicates thatthere exists convection in a sub-layer.
- In the lower viscosity region Roc remains a constant. In the moderateviscosity region as c increases Roc increases and reaches to a maximumvalue at $c = c^*$. The variation of c in the moderate viscosity ratiostabilizes the onset of oscillatory convection. For the values of c in thelarge viscosity ratio region as c increases $R_{0c}$ decreases. This interestingphenomenon shows that there exists convection in the sub-layer.
- The parameter $R_s > 0$ shows that as $R_s$ increases $R_{sc}$ increases, i.e.,the effect of $R_s > 0$stabilizes the onset of stationary convection.
- As the L increases, $R_{sc}$ decreases. This implies that the effect of $L$destabilizes the onset of stationary convection.
- As $R_s > 0$increases, $R_{oc}$ increases. This implies that the effect of$R_s > 0$stabilizes the onset of oscillatory convection.
- As $L$ increases $R_{oc}$ deceases, which indicates that the effect of $L$ destabilizes the convective system.
- As $P_r$ increases $R_{oc}$ decreases. This shows that the effect of $P_r$ destabilizes the onset of oscillatory convection.
- The maximum value of $z$ component of vertical velocity $W(z)$ on thecurve increases as $c$ increases. The maximum value of $W(z)$occurs atthe bottom warm boundary where the fluid is less viscous for a given$c$.
- The maximum value of $z$ component of temperature $\Theta(z)$on the curveincreases as $c$ increases. The maximum value of $\Theta(z)$occurs at thebottom warm boundary where the fluid is less viscous for a given $c$.This shows that temperature perturbations confine to low viscosityregion.
- Figure 12, clearly shows that $R_{oc} < R_{sc}$. This shows that, thermohaline convection with exponential variation of fluid property exhibitsoscillatory convection at the onset for finite$P_r$.

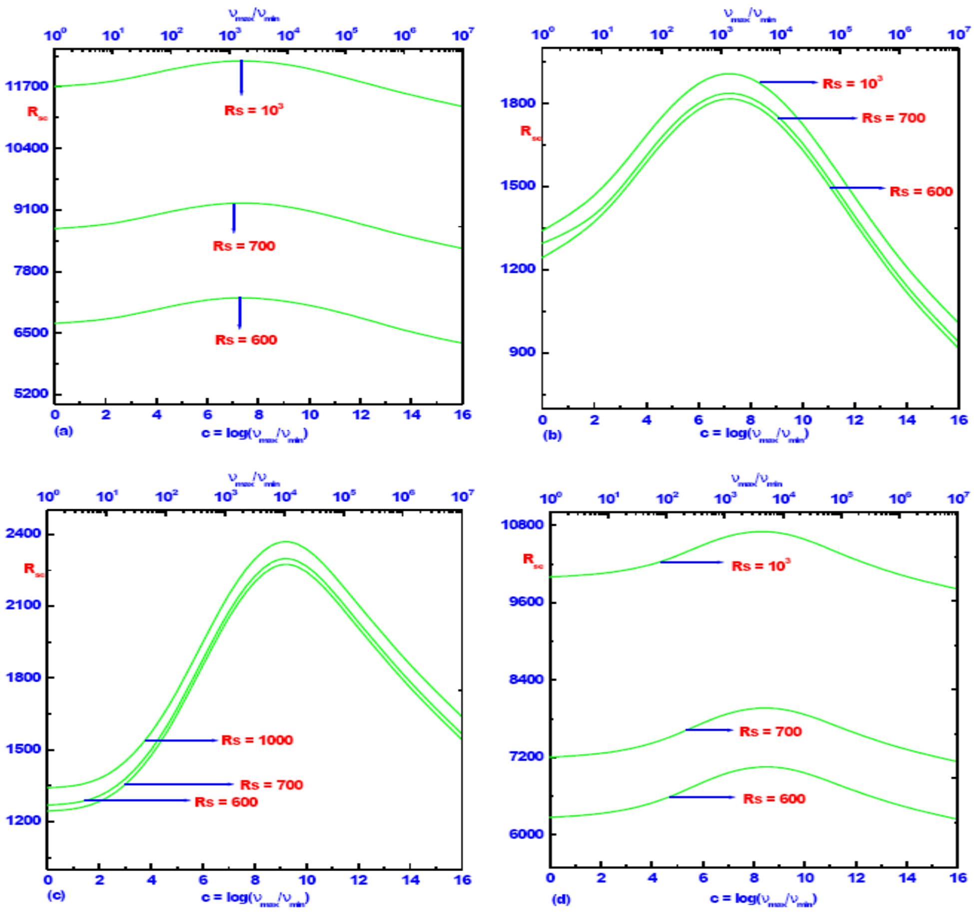


**Figure 1:**The effect of $R_{sc}$ and $c$ on $R_{sc}$ for $L$ = 0.1 and $P_r$.Numerically plotted neutral stability curves to represent onset of stationary convection for exponential fluid. Figure 1(a-d) are plotted for $R/R$, $R/F$, $F/R$ and $F/F$ boundary conditions respectively.

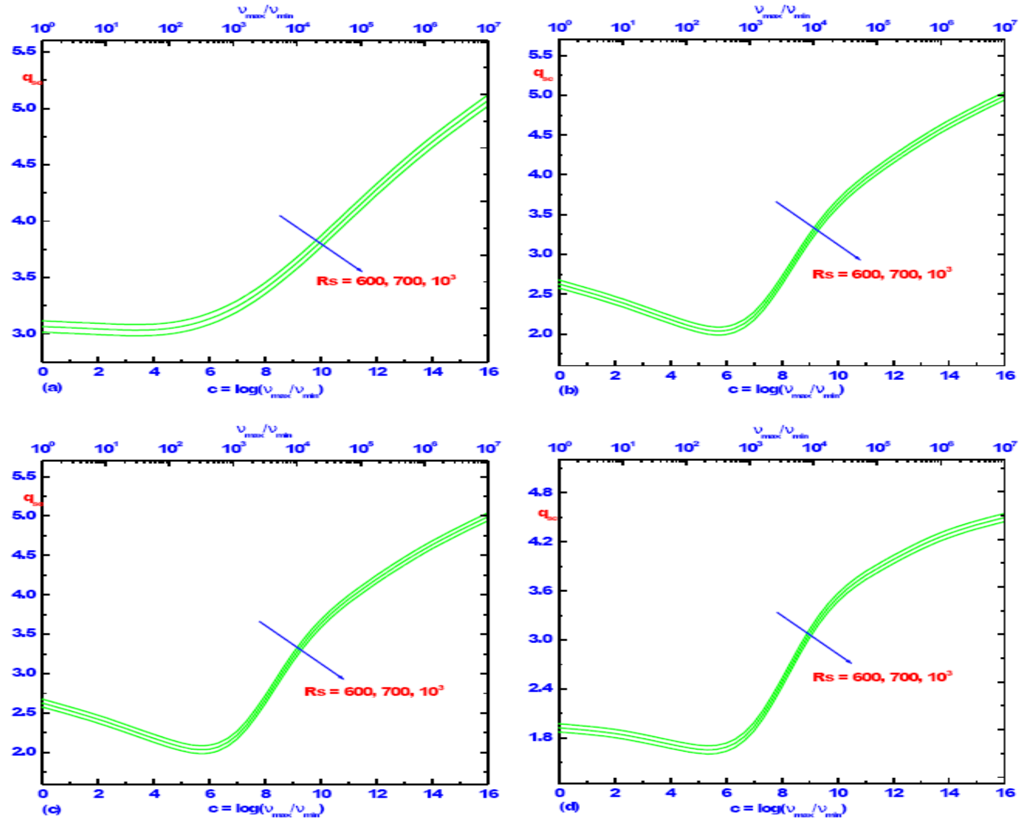


**Figure 2:**The effect of $R_s$ and $c$ on $q_{sc}$ for $L$ = 0.1 and $P_r$.Numerically plotted neutral stability curves to represent onset of stationary convection for exponential fluid. Figure 1(a-d) are plotted for $R/R$, $R/F$, $F/R$ and $F/F$ boundary conditions respectively.

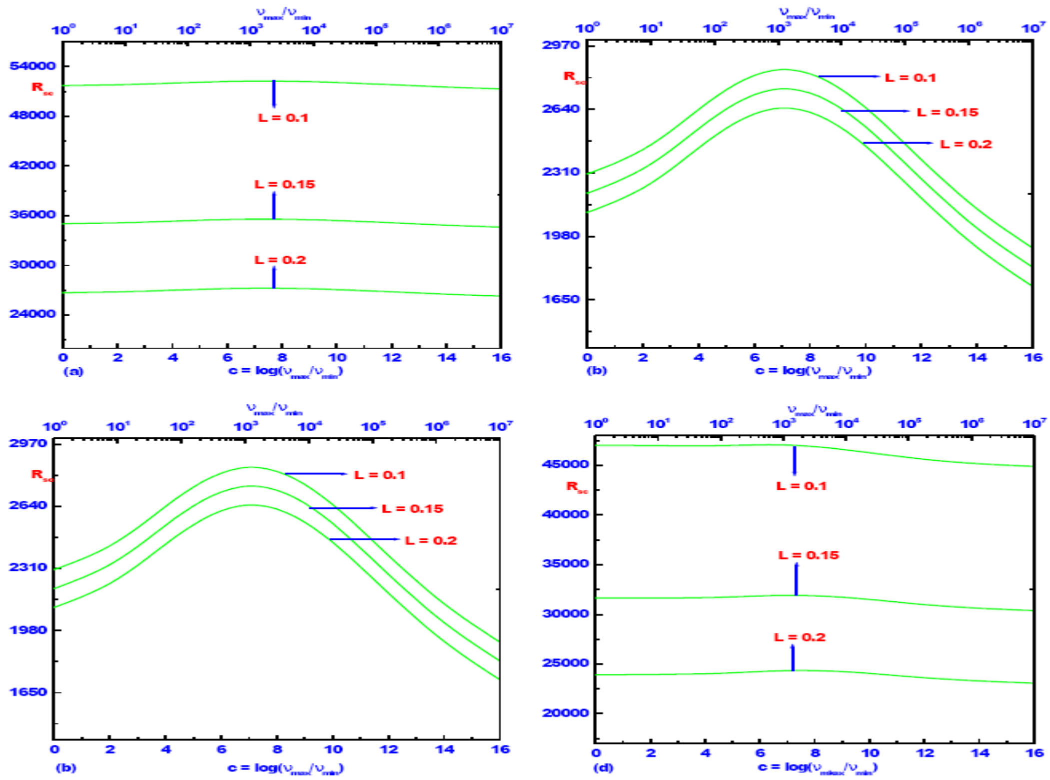


**Figure 3:**The effect of Lewis number $L$ and viscosity ratio $c$ on $R_{sc}$ for $R_s = 5 \times 10^3$.Numerically plotted neutral stability curves to represent onset of stationary convection for exponential fluid. Figure 3(a-d) are plotted for $R/R$, $R/F$, $F/R$ and $F/F$ boundary conditions respectively.

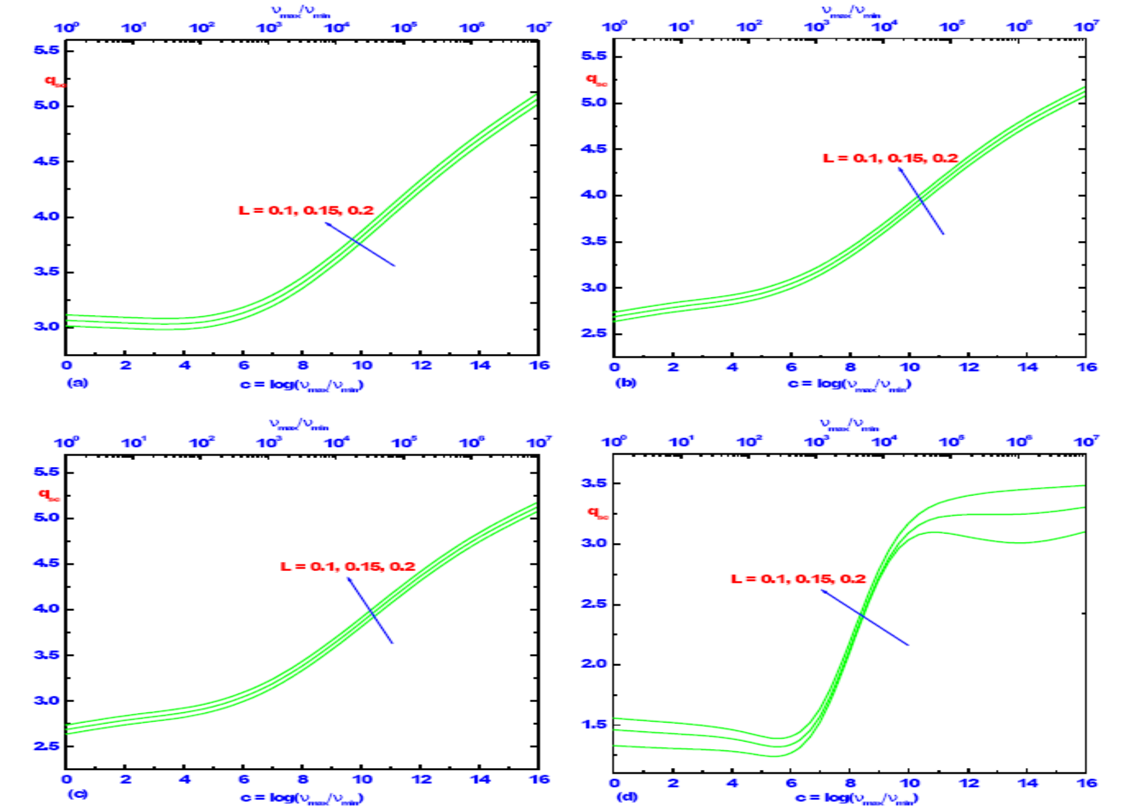


**Figure 4:**The effect of Lewis number $L$ and viscosity ratio $c$ on $q_{sc}$ for $R_s = 5 \times 10^3$.Numerically plotted neutral stability curves to represent onset of stationary convection for exponential fluid. Figure 4(a-d) are plotted for $R/R$, $R/F$, $F/R$ and $F/F$ boundary conditions respectively.

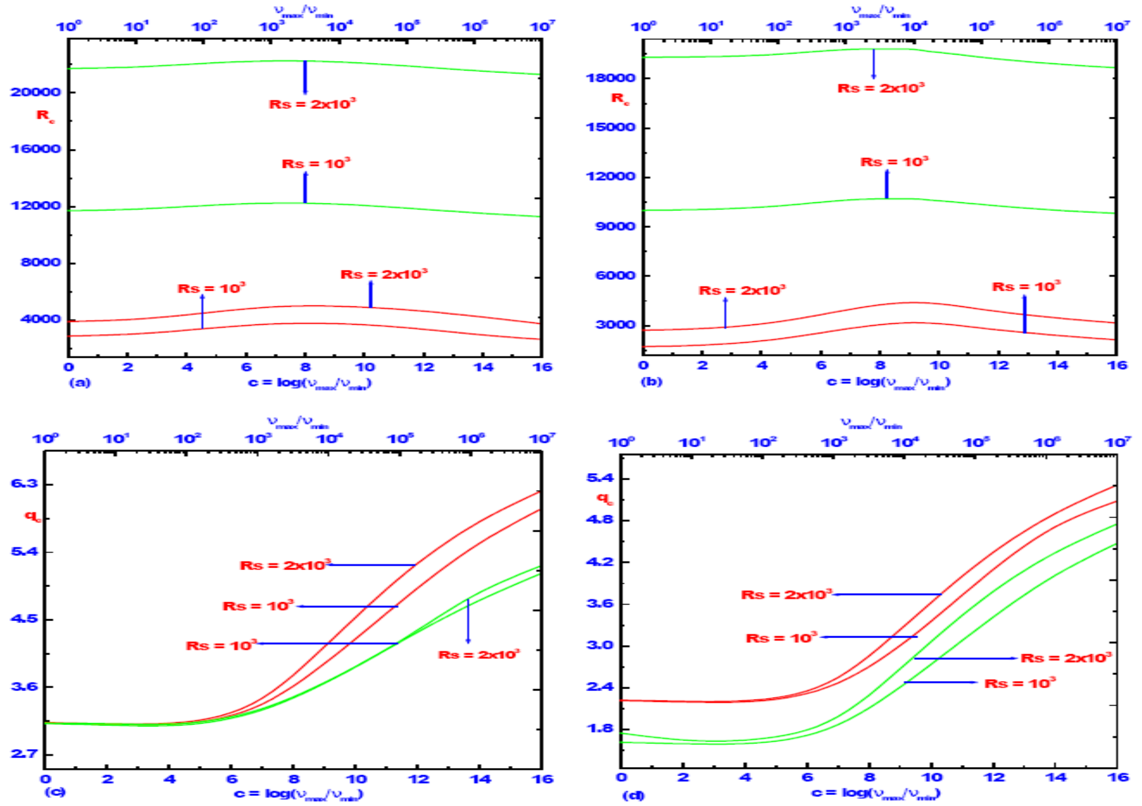


**Figure 5:**Critical Rayleigh number $R_c$ and critical wave number $q_c$ verses viscosity ratio $c$ for $L = 0.1$and $P_r = 10^3$. Figs. (a) and (c) are plotted for*R/R*boundaries. Figs. (b) and (d) are plotted for *F/F* boundary conditions. Green and red lines are corresponding to stationary and oscillatory convections, respectively.

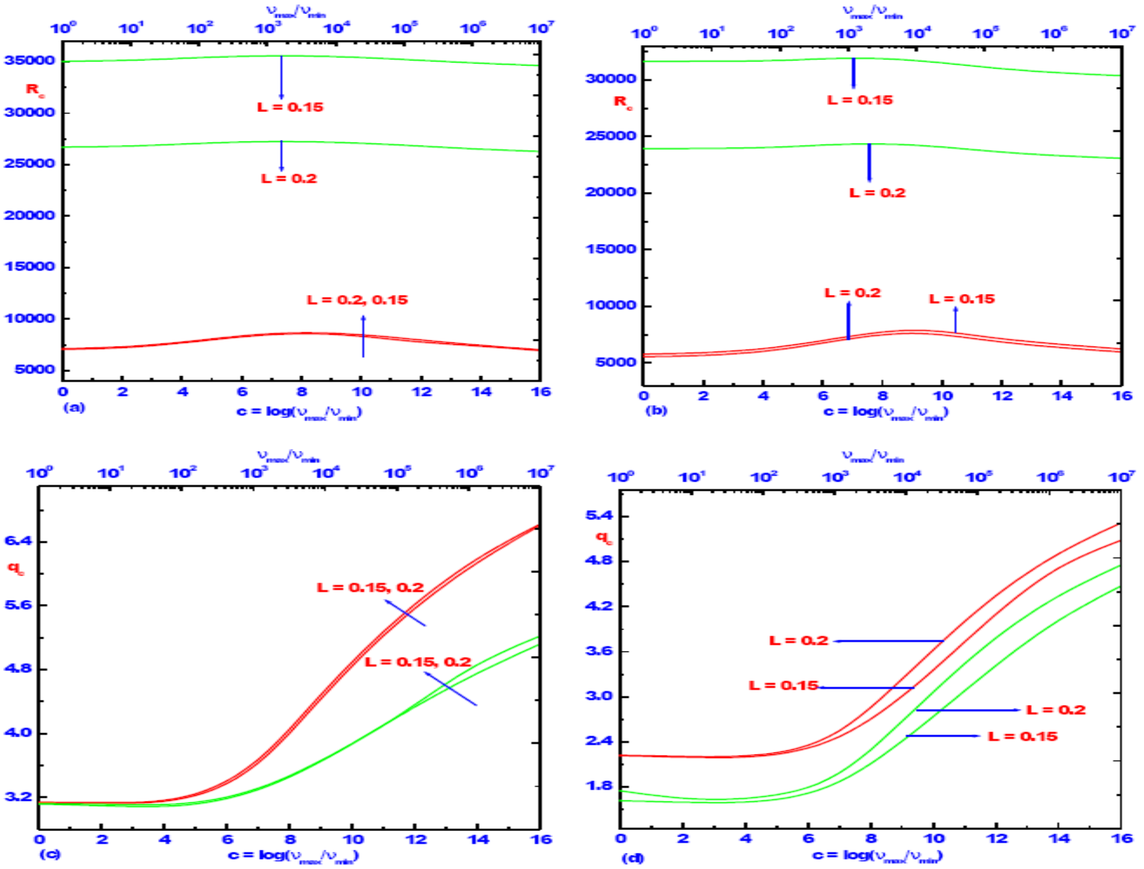


**Figure 6:**Critical Rayleigh number $R_c$ and critical wave number $q_c$ verses viscosity ratio $c$ for $R_s = 5 \times 10^3$ and $P_r = 10^3$. Figs. (a) and (c) are plotted for *R/R* boundaries. Figs. (b) and (d) are plotted for *F/F* boundary conditions. Green and red lines are corresponding to stationary and oscillatory convections, respectively.

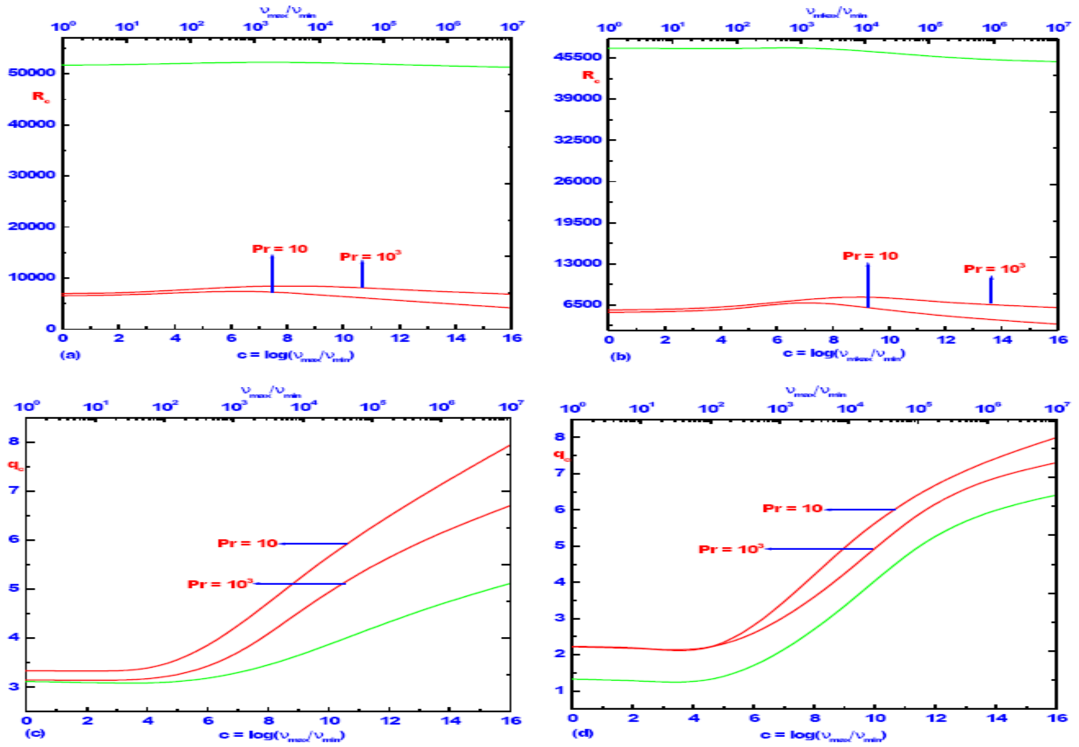


**Figure 7:**$P_r$ verses critical Rayleigh number and critical wave numbers for $R_s = 5 \times 10^3$ and $L = 0.1$. Figs. (a) and (c) are plotted for *R/R* boundaries. Figs. (b) and (d) are plotted for *F/F* boundary conditions. Green and red lines are correspond represent stationary and oscillatory convections, respectively.

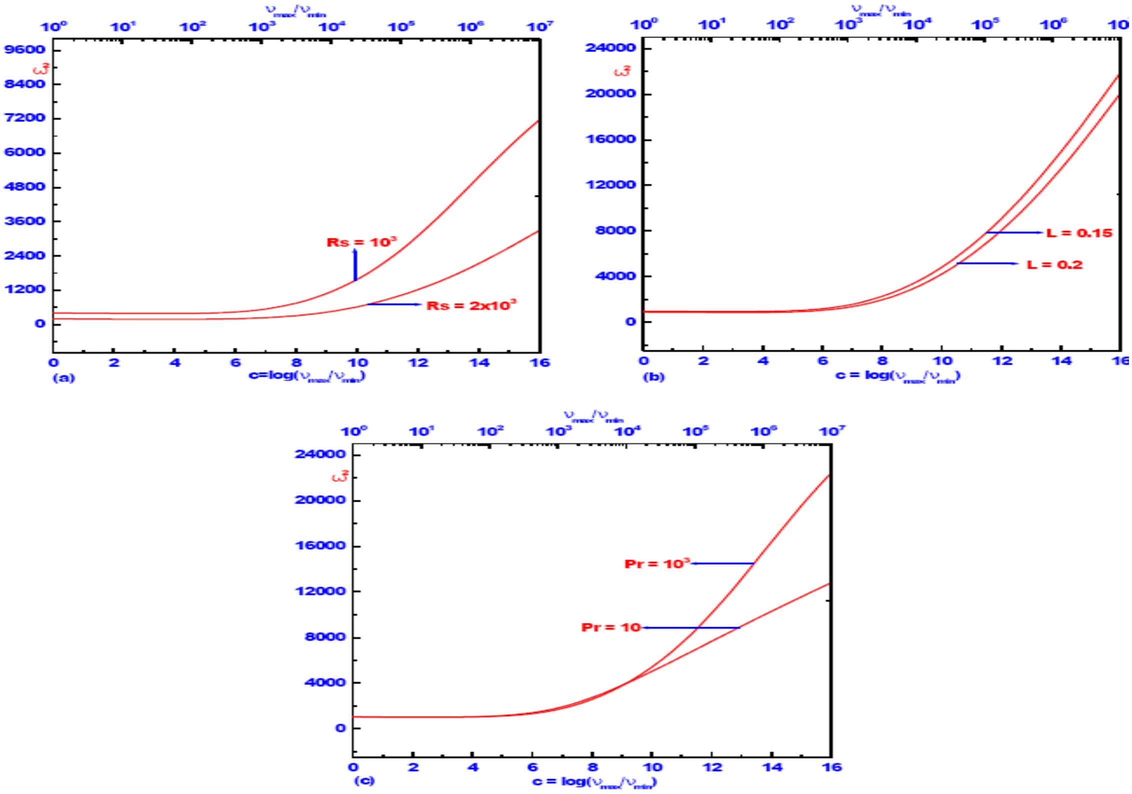


**Figure 8:** Until a moderate viscosity ratio $c$, frequency $w^2$ is almost constant. For large viscosity variation, $w^2$ increases. The frequency, $w^2$, decreases as $R_s$ and $L$ increases, while the opposite trend isobserved with respect to $P_r$.

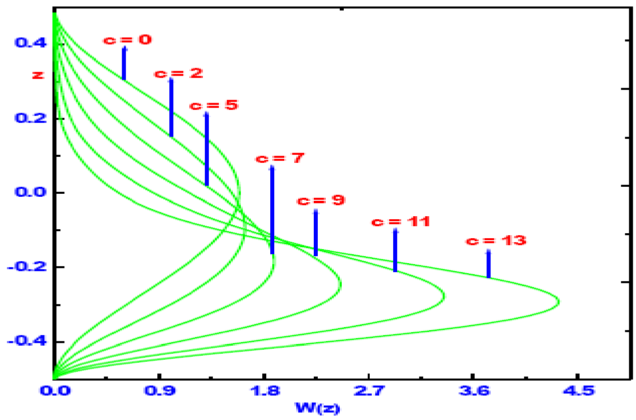


**Figure 9:**The vertical velocity eigen$W(z)$ functions for different c values for R/R boundary conditions with$L = 0.1,\ R_s = 5 \times 10^3$.

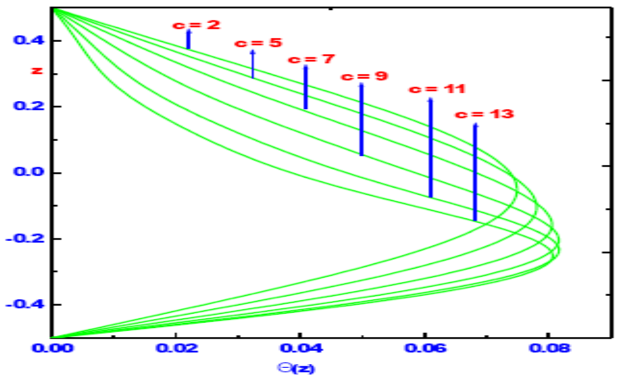


**Figure 10:**Temperature perturbation $\Theta(z)$ for different values of viscosity ration $c$ with $L = 0.1,\ R_s = 5 \times 10^3$and *R/R* boundary conditions.

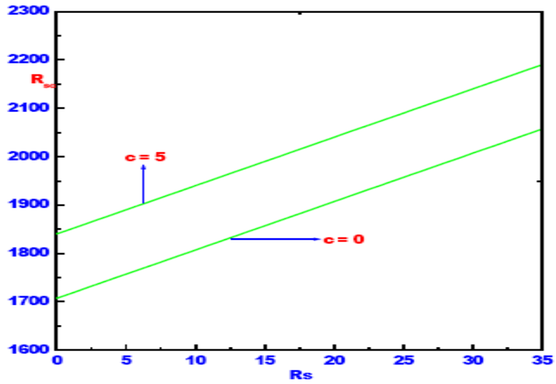


**Figure 11:**Effect of salinity Rayleigh number and viscosity variation c on critical Rayleigh number with $L = 0.1,\ R_s = 5 \times 10^3$ for stationary convection.

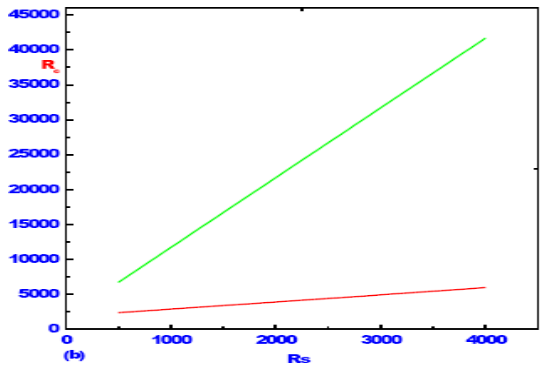


**Figure 12:** Effect of salinity Rayleigh number $c = 5$ on critical Rayleigh number with $= 0.1,\ R_s = 5 \times 10^3$, $P_r = 10^3$. Green line corresponding to stationary convection and red line corresponds to oscillatory convection.

## References


[1]. R.N.Horne and M.J. O'Sullevan, *Convection in a porous medium heated from below: the effect of temperature dependent viscosity and thermal expansion coefficient*, A.S.M.E.J. Heat Transfer, 100(3)(1978), pp.448.

[2]. S. Chandrashekar, *Hydrodynamic and Hydromagnatic stability*, Oxford: Clarendon. (1961).

[3]. D.D.Joseph, *Stability of fluid motions*, Vol.11. Springer, Berlin, (1976), pp. 274.

[4]. G.Verinis, *Effect of a stabilizing gradient of solute on thermal convection*, J.Fluid Mech., 34(1968), pp. 315.

[5]. H. Rubin, *Effect of nonlinear stabilizing salinity profiles on thermal convection in a porous medium layer*. Water Resour. Res., 9(1973), pp. 211.

[6]. H. Rubin, *Effect of solute dispersion on thermal convection in a porous medium layer*. Water Resour. Res., 9(1973), pp. 968.

[7]. D.A. Nield, *Onset of thermohaline convection in porous medium*. Water Resour. Res., 4(1968), pp. 553-560.

[8]. T.G.L. Shirtcliffe, *Thermosolutal convection; observation of an over-stable mode*, Nature (London), 213(1967), pp. 489.

[9]. J.S. Turner, *The behaviour of the stable salinity gradient heated from below*, J.Fluid Mech., 33(1968), pp. 183.

[10]. P.G. Bains and A.E.Gill, *Thermohaline convection with linear gradients*, J. Fluid Mech., 37(1969), pp. 289.

[11]. Torrence K. E, and Turcotte D. L, *Thermal convection with large viscosity variation*, J. Fluid Mech., 47(1971), pp.113.

[12]. Stengel K. C, Oliver D. S, and Booker J. R, *Onset of convection in a variable viscosity fluid*, J. Fluid Mech., 120(1982), pp. 411.

[13]. N.Rudraiah, I.S.Shivakumara and R.Fredcich, *The effect of rotation on linear and non-linear double-diffusive convection in a sparcely packed porous mediaum*, Int. J. Heat and Mass Tran., 29(1986), NO.9, pp. 1301.