Research Inventy: International Journal of Engineering And Science
Vol.6, Issue 2 (February 2016), PP -27-37
Issn (e): 2278-4721, Issn (p):2319-6483, www.researchinventy.com

Area relation of two right angled triangle in trigonometric form

Deshmukh Sachin Sandipan
Corps of Signals (Indian Army), state-Maharashtra, INDIA.

Abstract : *In this research paper ,explained trigomomatric area relation of two right angled triangle when sidemeasurement of both right angled triangle are equal. And that explanation given between base ,height ,hypogenous and area of right angled triangles with the help of formula.here be remember that, sidemeasurement of both right angled triangle are same.*

Keywords : *Area, Sidemeasurement, Relation, trigonometry, Right angled triangle.*

I. INTRODUCTION

Relation All Mathematics is a new field and the various relations shown in this research,
"Area relation of two right angled triangle in trigonometric form" is the one of the important research paper in the Relation All Mathematics and in future, any research related to this concept, that must be part of " Relation Mathematics " subject. Here, we have studied and shown new variables, letters, concepts, relations, and theorems. Inside the research paper, new concept of Trigonomatry about right angled triangle is explained.We have explained a new concept i.e. Sidemeasurement, which is very important related to 'Relation Mathematics' subject.
Here the relation between base ,height ,hypotenuse ,angle and area in two right angled triangle is explained in the form of trigonometry with the help of formula when the side-measurement of both the right angled triangles is same. here be remember that sidemeasurement of both right angled triangle are same.
In this "Relation All Mathematics" we have shown quadratic equation of rectangle. This "Relation All Mathematics" research work is near by 300 pages. This research is prepared considering the Agricultural sector mainly, but I am sure that it will also be helpful in other sector.

II. BASIC CONCEPT

2.1. Sidemeasurement(B) :-If sides of any geometrical figure are in right angle with each other , then those sides or considering one of the parallel and equal sides after adding them, the addition is the sidemeasurement .sidemeasurement indicated with letter 'B'
Sidemeasurement is a one of the most important concept and maximum base of the Relation All Mathematics depend apoun this concept.

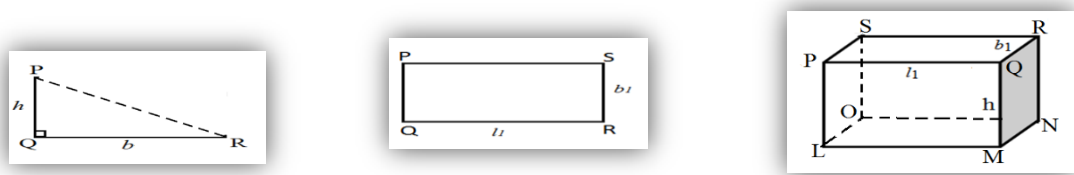

Figure I : Concept of sidemeasurement relation

I) Sidemeasurement of right angled triangle - B (ΔPQR) = $b + h$
In **Δ**PQR ,sides PQ and QR are right angle, performed to each other .

II) Sidemeasurement of rectangle-B(□PQRS)= $l_1 + b_1$
In □PQRS, opposite sides PQ and RS are similar to each other and m<Q = 90^o .here side PQ and QR are right angle performed to each other.

III) Sidemeasurement of cuboid–E_B(□PQRS) = $l_1 + b_1 + h_1$
In E(□PQRS),opposite sides are parallel to each other and QM are right angle performed to each other. Sidemeasurement of cuboid written as = E_B(□PQRS)

2.2)Important points of square-rectangle relation :-

I) For explanation of square and rectangle relation following variables are used

i) Area – A
ii) Perimeter – P
iii) Side measurement – B

II) For explanation of square and rectangle relation following letters are used

i) Area of square ABCD – A (□ABCD)
ii) Perimeter of square ABCD – P (□ABCD)
iii) Sidemeasurement of square ABCD – B (□ABCD)
iv) Area of rectangle PQRS – A (□PQRS)
v) Perimeter of rectangle PQRS – P (□PQRS)
vi) Sidemeasurement of rectangle PQRS – B (□PQRS)

II) For explanation of two right angled triangle relation, following letters are used

- In isosceles right angled triangle ΔABC [$45^0 – 45^0$- 90^0],side is assumed as 'l' and hypotenuse as 'X'
- In scalene right angled triangle ΔPQR[$\theta_1 – \theta_1'$ - 90^0] it`s base 'b_1' height 'h_1'and hypotenuse assumed as 'Y'
- In scalene right angled triangle ΔLMN[$\theta_2 – \theta_2'$ - 90^0]it`s base 'b_2' height 'h_2'an hypotenuse assumed as 'Z'

i) Area of isosceles right angled triangle ABC – A (ΔABC)
ii) Side-measurement of isosceles right angled triangle ABC – B (ΔABC)
iii) Area of scalene right angled triangle PQR – A (ΔPQR)
vi) Sidemeasurement of scalene right angled triangle PQR – B (ΔPQR)

2.3) Important Reference theorem of previous paper which used in this paper:-

Theorem : Basic theorem of area relation of isosceles right angled triangle and scalene right angled triangle.
The Sidemeasurement of isosceles right angled triangle and scalene right angled triangle is same then area of isosceles right angled triangle is more than area of scalene right angled triangle, at that time area of isosceles right angled triangle is equal to sum of the, area of scalene right angled triangle and Relation area formula of isosceles right angled triangle - scalene right angled triangle(K`) .

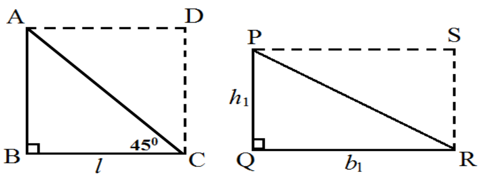

Figure II : Area relation of isosceles right angled triangle and scalene right angled triangle

Proof formula :- A(ΔABC) = A(ΔPQR) + $\frac{1}{2}\left[\frac{(b_1 + h_1)}{2} - h_1\right]^2$

[Note :- The proof of this formula given in previous paper and that available in reference**]**

Theorem : Basic theorem of sidemeasurement relation of isosceles right angled triangle and scalene right angled triangle.
Area of isosceles right angled triangle and scalene right angled triangle is same then sidemeasurement of scalene right angled triangle is more than sidemeasurement of isosceles right angled triangle , at that time sidemeasurement of scalene right angled triangle is equal to product of the, sidemeasurement of isosceles right angled triangle and Relation sidemeasurement formula of isosceles right angled triangle-scalene right angled triangle(V').

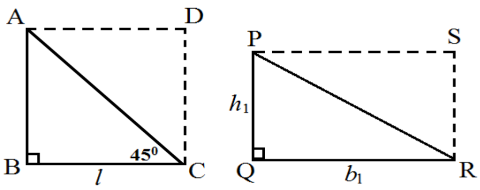

Figure III : Sidemeasurement relation of isosceles right angled triangle-scalene right angled triangle

Proof formula :- B(ΔPQR)= B(ΔABC) x $\frac{1}{2}\left[\frac{(n^2+1)}{n}\right]$

[Note :- The proof of this formula given in previous paper and that available in reference**]**

III) TRIGONOMETRIC RELATIONS

Relation –I: Proof of hypotenuse –area relation in isosceles right angled triangle and scalene right angled triangle .

Given :-In ΔABC ,m <A=45^0, m<B=90^0 and m <C=45^0
In ΔPQR ,m <P= θ_1, m<Q=90^0 and m<R= θ'_1
b_1 = Ycosθ_1 , h_1= Ysinθ_1 & $l^2/2 = X^2/4$
here , AB + BC = PQ + QR

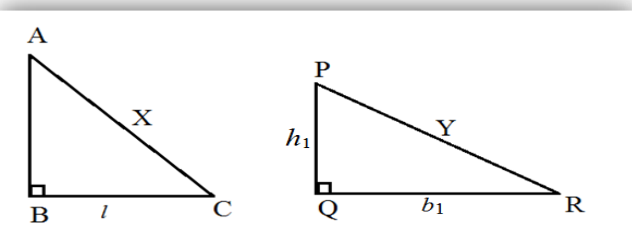

Figure IV: Trigonomatric hypotenuse –area relation in ΔABC ($45^0 – 45^0 - 90^0$) and ΔPQR(θ_1- $\theta'_1 – 90^0$)

To prove :- $X^2= Y^2/2\ (\sin 2\theta_1 + 1)$

Proof :- In ΔABC and ΔPQR ,

$$A(\Delta ABC) = A(\Delta PQR) + \frac{1}{2}\left[\frac{(b_1+h_1)}{2} - h_1\right]^2$$

…(Basic theorem of area relation of isosceles right angled triangle and scalene right angled triangle)

$$A(\Delta ABC) = A(\Delta PQR) + \frac{1}{2}\left[\frac{(b_1-h_1)}{2}\right]^2$$

$$\frac{l^2}{2} = \frac{b_1h_1}{2} + \frac{1}{2}\left[\frac{(b_1-h_1)}{2}\right]^2$$

$$\frac{l^2}{2} = \frac{(\mathrm{Ycos}\,\theta_1.\mathrm{Ysin}\,\theta_1)}{2} + \frac{1}{2}\left[\frac{(\mathrm{Ycos}\,\theta_1 - \mathrm{Ysin}\,\theta_1)}{2}\right]^2 \quad \text{… (Given)} \quad b_1 = \mathrm{Ycos}\theta\ ,\ h_1 = \mathrm{Ysin}\theta_1$$

$$\frac{l^2}{2} = \frac{Y^2(\sin\theta_1.\cos\theta_1)}{2} + \frac{Y^2(\cos\theta_1 - \sin\theta_1)^2}{8}$$

$$= \frac{Y^2(\sin\theta_1.\cos\theta_1)}{2} + \frac{Y^2}{8}(sin^2\theta_1 + cos^2\theta_1 - 2\sin\theta_1.\cos\theta_1) \quad \text{… } (a-b)^2=a^2 - b^2+2ab$$

$$= \frac{2.Y^2(\sin\theta_1.\cos\theta_1)}{4} + \frac{Y^2}{8}(1-2\sin\theta_1.\cos\theta)$$

$$sin^2\theta_1 + cos^2\theta_1 = 1$$

$$= \frac{2.Y^2(\sin\theta_1.\cos\theta_1)}{4} + \frac{Y^2}{8} - \frac{Y^2}{4}\sin\theta_1.\cos\theta_1$$

$$= \frac{Y^2(\sin\theta_1.\cos\theta_1)}{4} + \frac{Y^2}{8}$$

but in Fig VI , $\frac{l^2}{2} = \frac{X^2}{4}$ …(Given)

$$\frac{X^2}{4} = \frac{Y^2(\sin\theta_1.\cos\theta_1)}{4} + \frac{Y^2}{8}$$

$$\frac{X^2}{4} = \frac{Y^2(\sin\theta_1.\cos\theta_1)}{4} + \frac{Y^2}{8}$$

$X^2 = Y^2\, sin\theta_1.cos\theta_1 + \frac{Y^2}{2}$

$X^2 = \frac{Y^2}{2}\; 2\sin\theta_1.\cos\theta_1 + \frac{Y^2}{2}$

$X^2 = \frac{Y^2 \sin 2\theta_1}{2} + \frac{Y^2}{2}$

$\ldots\ \sin2\theta = 2\sin\theta_1.\cos\theta_1$

$X^2 = \frac{Y^2}{2}(\sin2\theta_1 + 1)$ ... $\sin2\theta \le 1$

Hence , we have Prove that , Proof of hypotenuse –area relation in isosceles right angled triangle and scalene right angled triangle.
This formula clear that ,when given the sidemeasurement and angle of any right angled triangle then we can be find hypogenous of the right angled triangle .

Example:-

The sidemeasurement and angle of a right angled triangle ΔPQR is 20 cm and 30^o respectively .then find the hypotenious that right angled triangle?

Ans :-

Given – Side measurement of ΔPQR = 20 cm

Angle of ΔPQR = 30^o

To find :- hypotenious of ΔPQR = ?

Formula :

$X^2 = \frac{Y^2}{2}(\sin2\theta_1 + 1)$

Side of ΔABC = l $= \frac{PQ+QR}{2} = \frac{20}{2} = 10$

$X^2 = 10^2 + 10^2 = 200$

$200 = \frac{Y^2}{2}(\sin 2x30) + 1)$

$200 = \frac{Y^2}{2}(\sin 60 + 1)$

$200 = \frac{Y^2}{2}(1.8660)$

$\frac{400}{(1.8660)} = Y^2$

$Y^2 = 214.3593539$

$Y = 14.64101615$

Hence the hypotenious of right angled triangle ΔPQR is 14.64101615 cm .

Relation –II: Proof of hypotenuse –area relation in two scalene right angled triangles

Given :-In ΔABC , m<A=45^0, <B=90^0 and m<C=45^0

In ΔPQR , $m<P= \theta_1$, $<Q=90^0$ and $m<R= \theta'_1$
In ΔLMN ,$m<P= \theta_2$, $<Q=90^0$ and $m <R= \theta'_2$
here , $AB + BC = PQ + QR= LM + MN$

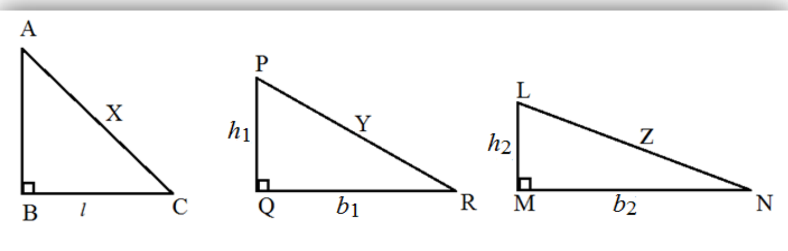

Figure V : Trigonomatric hypotenuse –area relation in $\Delta PQR(\theta_1 - \theta'_1 - 90^0)$ and ΔLMN $(\theta_2 - \theta'_2 - 90^0)$

To prove :- $\frac{Y^2}{Z^2} = \frac{(\sin 2\theta_2 + 1)}{(\sin 2\theta_1 + 1)}$

Proof :- In ΔABC and ΔPQR ,

$X^2=\frac{Y^2}{2}(\sin 2\theta_1 + 1)$ …(i)

…(Proof of hypotenuse –area relation in isosceles right angled triangle and scalene right angled)

In ΔABC and ΔLMN ,

$X^2=\frac{Z^2}{2}(\sin 2\theta_2 + 1)$ …(ii)

… (Proof of hypotenuse –area relation in isosceles right angled triangle and scalene right angled)

$$\frac{Y^2}{2}(\sin 2\theta_1 + 1) = \frac{Z^2}{2}(\sin 2\theta_2 + 1) \quad \text{… From equation (i) and (ii)}$$

$$\frac{Y^2}{Z^2} = \frac{(\sin 2\theta_2 + 1)}{(\sin 2\theta_1 + 1)}$$

Hence , we have Prove that , Proof of hypotenuse –area relation in two scalene right angled triangles**.**
This formula clear that ,when given the hypogenous and angle of any right angled triangle then we can be find angle of another right angled triangle when given the hypotenious and vise varsa which both right angled triangle are sidemeasurement is equal but not required to known .

Relation –III: Proof of base –area relation in isosceles right angled triangle and scalene right angled triangle
Given :-In ΔABC , $m<A=45^0$, $m<B=90^0$ and $m<C=45^0$
In ΔPQR , $m<P= \theta_1$, $m<Q=90^0$ and $m<R= \theta'_1$
$b_1 = Y\cos\theta_1$, $h_1 = Y\sin\theta_1$ & $l^2 / 2 = X^2 / 4$
here , $AB + BC = PQ + QR$

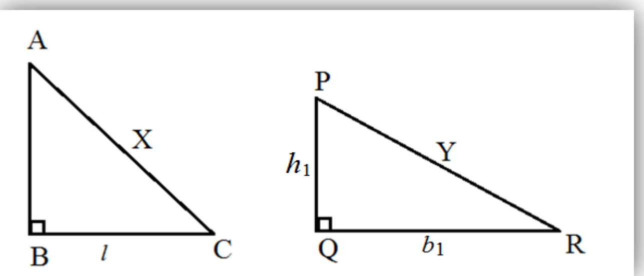

Figure VI : Trigonomatric base –area relation in ΔABC $(45^0 - 45^0 - 90^0)$ and $\Delta PQR(\theta_1 - \theta'_1 - 90^0)$

To prove :- $l^2 = b_1^2 \; \frac{1}{4}(1+\tan\theta_1)^2$

Proof :- In ΔABC and ΔPQR,

$X^2 = \frac{Y^2}{2}(\sin 2\theta_1 + 1)$

… (Proof of hypotenuse –area relation in isosceles right angled triangle and scalene right angled)

$X^2 = \frac{Y^2}{2}(\sin 2\theta_1 + 1)$

$2\,l^2 = \frac{b_1^2}{\cos^2\theta 1} \; \frac{(\sin 2\theta 1 + 1)}{2}$ …($X = \sqrt{2}\,l^2$, $Y = b_1/\cos\theta_1$)

$l^2 = \frac{b_1^2(\sin 2\theta 1 + 1)}{4.\cos^2\theta 1}$

$l^2 = \frac{b_1^2 .2\sin\theta 1.\cos\theta 1}{4.\cos^2\theta 1} + \frac{\sin^2\theta 1}{4.\cos^2\theta 1} + \frac{\cos^2\theta 1}{4.\cos^2\theta 1}$

$= \frac{b_1^2.\ 2\sin\theta 1}{4.\cos^2\theta 1} + \frac{\sin^2\theta 1}{4.\cos^2\theta 1} + \frac{1}{4}$

$= b_1^2 \; [\frac{2}{4}\tan\theta_1] + [\frac{1}{4}\tan^2\theta_1] + [\frac{1}{4}]$

$= b_1^2 \; \frac{1}{4} \; [2\tan\theta_1] + [\tan^2\theta_1] + [1]$

$l^2 = b_1^2 \; \frac{1}{4}(1+\tan\theta_1)^2$

Hence , we have Prove that , Proof of base –area relation in isosceles right angled triangle and scalene right angled triangle.
This formula clear that ,when given the sidemeasurement and angle of any right angled triangle then we can be find base of the right angled triangle .

Relation –IV: Proof of base –area relation in two scalene right angled triangles
Given :-In ΔABC , m<A=45^0, m<B=90^0 and m<C=45^0
In ΔPQR , m<P= θ_1, m<Q=90^0 and m<R= θ'_1
In ΔLMN ,m<P= θ_2, m<Q=90^0 and m <R= θ'_2
here , AB + BC = PQ + QR= LM + MN

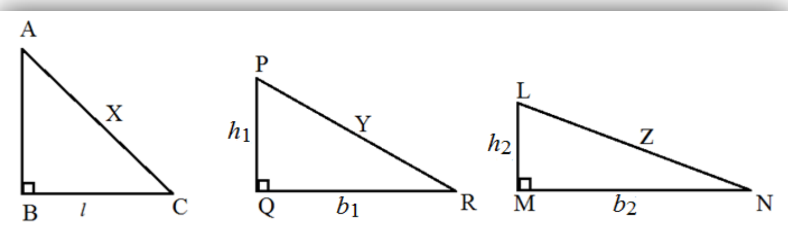

Figure VII : Trigonomatric base –area relation in ΔPQR(θ_1- θ'_1 – 90^0) and ΔLMN (θ_2- θ'_2 – 90^0)

To prove :- $b_1^2 = b_2^2 \; \frac{(1+\tan\theta 2)^2}{(1+\tan\theta 1)^2}$

Proof :- In ΔABC and ΔPQR ,

$l^2 = b_1^2 \frac{1}{4}(1+\tan\theta_1)^2$ …(i)

… (Proof of base –area relation in isosceles right angled triangle and scalene right angled)

In ΔABC and ΔLMN ,

$l^2 = b_2^2 \frac{1}{4}(1+\tan\theta_2)^2$ …(ii)

…(Proof of base –area relation in isosceles right angled triangle and scalene right angled)

$b_1^2 \frac{1}{4}(1+\tan\theta_1)^2 = b_2^2 \frac{1}{4}(1+\tan\theta_2)^2$ … From equation no (i) and (ii)

$b_1^2 = b_2^2 \frac{(1+\tan\theta 2)^2}{(1+\tan\theta 1)^2}$

Hence , we have Prove that , Proof of base –area relation in two scalene right angled triangles.
This formula clear that ,when given the base and angle of any right angled triangle then we can be find angle of another right angled triangle when given the base and vise varsa which both right angled triangle are sidemeasurement is equal but not required to known .

Relation –V: Proof of height –area relation in isosceles right angled triangle and scalene right angled triangle

Given :-In ΔABC , m<A=45^0, m<B=90^0 and m<C=45^0
In ΔPQR , m<P= θ_1, m<Q=90^0 and m<R= θ'_1
b_1 = Y$\cos\theta_1$, h_1= Y$\sin\theta_1$
here , AB + BC = PQ + QR

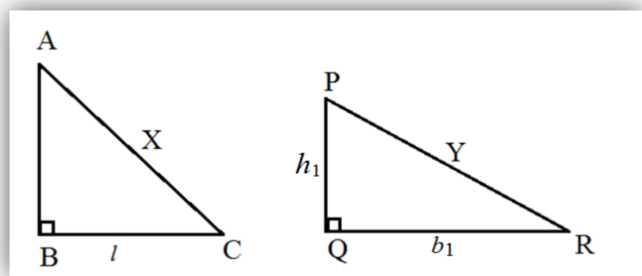

Figure VIII : Trigonomatric height –area relation in ΔABC ($45^0 – 45^0$ - 90^0) and ΔPQR(θ_1- θ'_1 – 90^0)

To prove :- $l^2 = h_1^2 \frac{1}{4}(1+\cot\theta_1)^2$

Proof :- In ΔABC and ΔPQR ,

. . $X^2 = \frac{Y^2}{2}(\sin 2\theta_1 + 1)$ …(i)

…(Proof of hypotenuse –area relation in isosceles right angled triangle and scalene right angled)

$2\,l^2 = \frac{h_1^2}{\sin^2\theta 1} \frac{(\sin 2\theta 1 + 1)}{2}$ …($X=\sqrt{2}l^2$, $Y=\frac{h_1}{\sin\theta 1}$)

$l^2 = \frac{h_1^2 (\sin 2\theta 1 + 1)}{4 .\sin^2\theta 1}$

$l^2 = \frac{h_1^2 (\sin 2\theta 1 + 1)}{4 .\sin^2\theta 1}$

$l^2 = \frac{h_1^2 .2\sin\theta 1.\cos\theta 1}{4.sin^2\theta 1} + \frac{sin^2\theta 1}{4.sin^2\theta 1} + \frac{cos^2\theta 1}{4.sin^2\theta 1}$

$l^2 = h_1^2 \ (1 + 2\cot\theta 1. cot^2\theta 1] + \frac{cos^2\theta 1}{4.sin^2\theta 1}$

$= h_1^2 \ [\frac{2}{4}\tan\theta_1] + [\frac{1}{4} \ tan^2\theta_1] + [\frac{1}{4}]$

$= h_1^2 \ \frac{1}{4} \ [2\tan\theta_1] + [tan^2\theta_1] + [1]$

$$l^2 = h_1^2 \ \frac{1}{4}(1 + \tan\theta_1)^2$$

Hence , we have Prove that , Proof of height –area relation in isosceles right angled triangle and scalene right angled triangle .
This formula clear that ,when given the sidemeasurement and angle of any right angled triangle then we can be find height of the right angled triangle .

Relation –VI: Proof of height –area relation in two scalene right angled triangles
Given :-In ΔABC , $m\angle A=45^0$, $m\angle B=90^0$ and $m\angle C=45^0$
In ΔPQR , $m\angle P= \theta_1$, $m\angle Q=90^0$ and $m\angle R= \theta'_1$
In ΔLMN ,$m\angle P= \theta_2$, $m\angle Q=90^0$ and $m\angle R= \theta'_2$
here , $AB + BC = PQ + QR$

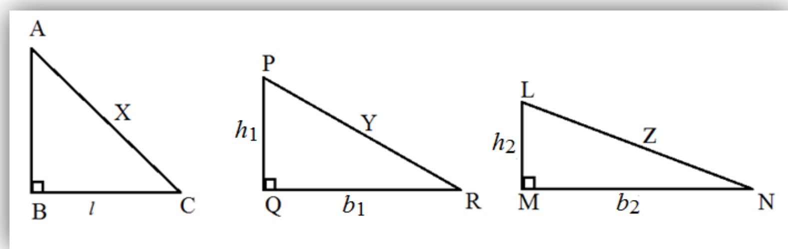

Figure IX : Trigonomatric height –area relation in $\Delta PQR(\theta_1 - \theta'_1 - 90^0)$ and $\Delta LMN\ (\theta_2 - \theta'_2 - 90^0)$

To prove :- $$h_1^2 = h_2^2 \ \frac{(1+\tan\theta_1)2}{(1+\tan\theta_2)2}$$

Proof :- In ΔABC and ΔPQR ,

$l^2 = h_1^2 \frac{1}{4}(1+ \cot\theta_1)^2$ …(i)

…(Proof of height –area relation in isosceles right angled triangle and scalene right angled)

In ΔABC and ΔLMN ,

$l^2 = h_2^2 \frac{1}{4}(1+ \cot\theta_2)^2$ …(ii)

…(Proof of height –area relation in isosceles right angled triangle and scalene right angled)

$h_1^2 \frac{1}{4}(1+ \cot\theta_1)^2 = h_2^2 \frac{1}{4}(1+ \cot\theta_2)^2$ From equation no (i) and (ii)

$$h_1^2 = h_2^2 \ \frac{(1+\cot\theta_2)2}{(1+\cot\theta_1)2}$$

$$h_1^2 = h_2^2 \ \frac{(1+\tan\theta_1)2}{(1+\tan\theta_2)2}$$

Hence , we have Prove that , Proof of height –area relation in two scalene right angled triangles .
This formula clear that ,when given the height and angle of any right angled triangle then we can be find angle of another right angled triangle when given the height and vise varsa which both right angled triangle are sidemeasurement is equal but not required to known .

Relation –VII: Proof of base –height area relation in two scalene right angled triangles
Given :-In ΔABC , $m\angle A=45^0$, $m\angle B=90^0$ and $m\angle C=45^0$
In ΔPQR , $m\angle P= \theta_1$, $m\angle Q=90^0$ and $m\angle R= \theta'_1$
In ΔLMN ,$m\angle P= \theta_2$, $m\angle Q=90^0$ and $m\angle R= \theta'_2$
here , $AB + BC = PQ + QR= LM + MN$

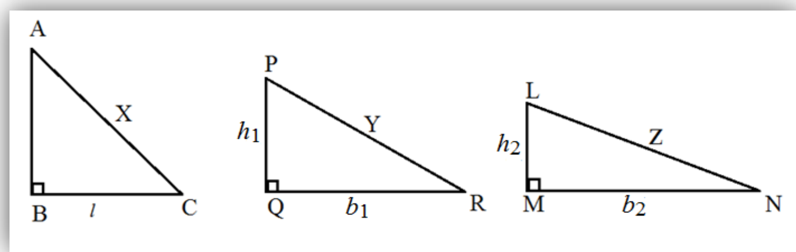

Figure X : Trigonomatric base –height area relation in $\Delta PQR(\theta_1 - \theta'_1 - 90^0)$ and ΔLMN $(\theta_2 - \theta'_2 - 90^0)$

To prove :- $b_1^2 = h_2^2 \frac{(1+\cot\theta_2)2}{(1+\tan\theta_1)2}$

Proof :- In ΔABC and ΔPQR ,

$l^2 = b_1^2 \frac{1}{4}(1+\tan\theta_1)^2$ …(i)

…(Proof of base –area relation in isosceles right angled triangle and scalene right angled)

$l^2 = h_1^2 \frac{1}{4}(1+\cot\theta_1)^2$ …(ii)

…(Proof of height –area relation in isosceles right angled triangle and scalene right angled)

$b_1^2 \frac{1}{4}(1+\tan\theta_1)^2 = h_2^2 \frac{1}{4}(1+\cot\theta_2)$ … From equation no (i) and (ii)

$b_1^2 = h_2^2 \frac{(1+\cot\theta_2)2}{(1+\tan\theta_1)2}$

Hence , we have Prove that , Proof of base –height area relation in two scalene right angled triangles. This formula clear that ,when given the base and angle of any right angled triangle then we can be find angle of another right angled triangle when given the height and vise varsa which both right angled triangle are sidemeasurement is equal but not required to known .

Relation –VIII: Proof of area relation in isosceles right angled triangle and scalene right angled triangle
Given :-In ΔABC , $m<A=45^0$, $m<B=90^0$ and $m<C=45^0$
In ΔPQR , $m<P=\theta_1$, $m<Q=90^0$ and $m<R=\theta'_1$
here , AB + BC = PQ + QR

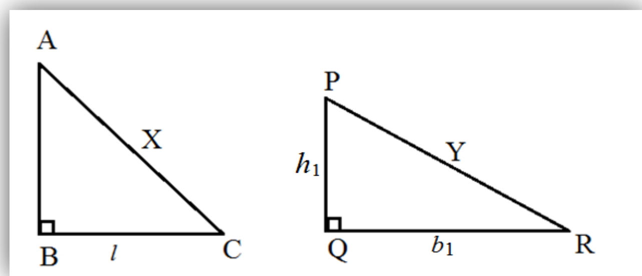

Figure XI : Trigonomatric area relation in ΔABC $(45^0 - 45^0 - 90^0)$ and $\Delta PQR(\theta_1 - \theta'_1 - 90^0)$

To prove :- $A(\Delta ABC) = A(\Delta PQR) \times \frac{(1+\tan\theta 1)\text{x}(1+\cot\theta 1)}{4}$

Proof :- In ΔABC and ΔPQR ,

$l^2 = b_1^2 \frac{1}{4}(1+\tan\theta_1)^2$ …(i)

…(Proof of base –area relation in isosceles right angled triangle and scalene right angled)

$l = b_1 \frac{1}{2}(1+\tan\theta_1)$

$$\frac{1}{2}\ l^2 = \frac{1}{2}\ \frac{b_1 h_1}{h_1}\ \text{x}\ \frac{(1+\tan\theta 1)}{2}$$

$A(\Delta ABC) = A(\Delta PQR)\ \frac{1}{h_1}\ \text{x} \frac{(1+\tan\theta 1)}{2}$

But , $l = h_1 . \frac{(1+\cot\theta 1)}{2}$

$A(\Delta ABC) = A(\Delta PQR)\ \text{x}\ \frac{h_1(1+\cot\theta 1)}{2h_1}\ \text{x}\ \frac{(1+\tan\theta 1)}{2}$

$(\Delta ABC) = A(\Delta PQR)\ \text{x}\ \frac{(1+\tan\theta 1)\text{x}\ (1+\cot\theta 1)}{4}$

Hence , we have Prove that , Proof of area relation in isosceles right angled triangle and scalene right angled triangle .
This formula clear that ,when given the sidemeasurement and angle of any right angled triangle then we can be find area of the right angled triangle .

Relation –IX: Proof of area relation in two scalene right angled triangles
Given :-In ΔABC , m<A=45^0, m<B=90^0 and m <C=45^0
In ΔPQR , m<P= θ_1,m <Q=90^0 and m<R= θ'_1
In ΔLMN ,m<P= θ_2, m<Q=90^0 and m<R= θ'_2
here , AB + BC = PQ + QR= LM + MN

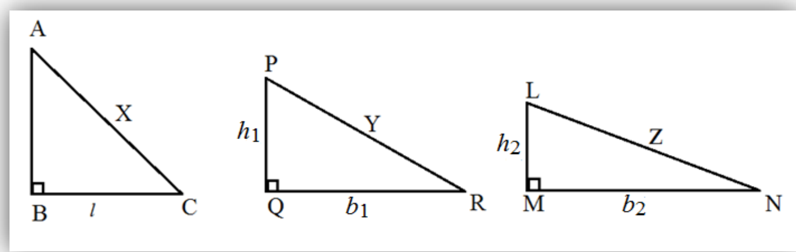

Figure XII : Trigonomatric area relation in $\Delta PQR(\theta_1 - \theta'_1 - 90^0)$ and $\Delta LMN\ (\theta_2 - \theta'_2 - 90^0)$

To prove :- $A(\Delta PQR) = A(\Delta LMN)\ \text{x}\ \frac{(1+\tan\theta 2)\ \text{x}\ (1+\cot\theta 2)}{(1+\tan\theta 1)\ \text{x}\ (1+\cot\theta 1)}$

Proof :- In ΔABC and ΔPQR ,

$(\Delta ABC) = A(\Delta PQR)\ \text{x}\ \frac{(1+\tan\theta 1)\text{x}\ (1+\cot\theta 1)}{4}$ …(i)

…(Proof of area relation in isosceles right angled triangle and scalene right angled triangle)

$(\Delta ABC) = A(\Delta LMN)\ \text{x}\ \frac{(1+\tan\theta 2)\text{x}\ (1+\cot\theta 2)}{4}$ …(ii)

…(Proof of area relation in isosceles right angled triangle and scalene right angled triangle)

$$A(\Delta PQR)\ \text{x}\ \frac{(1+\tan\theta 1)\text{x}\ (1+\cot\theta 1)}{4} = A(\Delta LMN)\ \text{x}\ \frac{(1+\tan\theta 2)\text{x}\ (1+\cot\theta 2)}{4}$$

$$A(\Delta PQR) = A(\Delta LMN\ \text{x}\ \frac{(1+\tan\theta 2)\ \text{x}\ (1+\cot\theta 2)}{(1+\tan\theta 1)\ \text{x}\ (1+\cot\theta 1)}$$

Hence , we have Prove that , Proof of area relation in two scalene right angled triangles **.**
This formula clear that ,when given the area and angle of any right angled triangle then we can be find angle of another right angled triangle when given the area and vise varsa which both right angled triangle are sidemeasurement is equal but not required to known .

IV. CONCLUTION

"Area relation of two right angled triangle in trigonometric form" this research article conclude that Trigonomatric area relation between two right angled triangle explained with the help of trigonometric equations, when area of both are equal.

V. ACKNOWLEDGEMENT

The present invention is published by with support of Corps of Signals (Indian Army).
Also the author would like to thank the anonymous referee(s) for their careful checking of the details , Valuable suggestions and comments that improved this research paper.

References

[1]. Surrounding agricultural life.
[2]. Answers.yahoo.com (www.answers.yahoo.com)
[3]. Wikipedia , Google and other search engines. (www.wikipedia.org)
[4]. Mr.Deshmukh Sachin Sandipan , "Area and perimeter relation of square and rectangle(Relation All Mathematics)" DOI:10.9790/5728-10660107 /IOSR-JM-A010660107
[5]. Mr.Deshmukh Sachin Sandipan , "Area and sidemeasurement relation of two right angled triagnel(Relation All Mathematics)" DOI:10.9790/5728-11220513 /IOSR-JM-B011220513
[6]. Mr.Deshmukh Sachin Sandipan , "Volume mensuration relation of two cuboids (Relation All Mathematics)" DOI:10.14445/22315373/IJMTT-V19P515
[7]. Mr.Deshmukh Sachin Sandipan , "Surface area mensuration relation of two cuboids (Relation All Mathematics)" DOI:10.14445/22315373/IJMTT-V23P502
[8]. Mr.Deshmukh Sachin Sandipan , "Concept of quadratic equation of right angled triangle to relation all mathematics method)"IJMR-Vol 7,Num 1(2015).pp.75-90
[9]. Mr.Deshmukh Sachin Sandipan, "The great method of Relation all Mathematics"[Marathi Book](http://www.readwhere.com/publication/9916/Relation-All-Mathematics-)