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Probabilistic Fatigue Risk Management of Aircraft Components

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Abstract : *A double cantilever beam (DCB) made of sandwich composite materials was probabilistically assessed for fatigue management with a maintenance plan. The probabilistic assessment is based on a probabilistic algorithm RPI (Recursive Probability Integration) for fleet risk management. RPI combines random simulations with numerical integration which maintains the generality of random simulations while taking advantage of the efficiency feature of numerical integration. An initial debond (crack) length is assumed to exist at the upper face/core interface and the crack remains at or near this interface during the crack propagation phase, and the risk of failure is monitored and managed by nondestructive inspections and subsequent maintenance actions. An empirical energy-release rate equation of the sandwich DCB based on a modified beam theory was adopted for this study. It is assumed that when the Mode-I stress-intensity factor exceeds the fracture toughness, the stress will cause unstable crack growth and structural failure. Probabilistic distributions are used to treat uncertainties in fracture toughness, crack growth rate, manufacturing and repair quality. Maximum flight load is modeled as an independent random process. The study has demonstrated that RPI can be several orders of magnitude more efficient than the traditional Monte Carlo method for aircraft fleet risk management.*

Keywords: *damage tolerance, fatigue, nondestructive inspection, probability of failure, risk*

I. Introduction

Sandwich composite structures can achieve extremely low weight and high flexural stiffness in aerospace applications. However, the disbond between the facesheet and the core requires special attention and further study. A face sheet to core separation and core fracture propagation due to the GAG (Ground-Air-Ground) fatigue cycles can lead to a significant reduction of the structural capability. Due to the complexity of the build-up and complicated manufacturing methods, sandwich structures could contain defects (initial damage) of various kinds (debonds, gaps, etc.). Moreover, during their operational lifetime they are sometimes subjected to low/moderate impacts and commonly subjected to long term cyclic load (fatigue). As a result, damage may be initiated with different sizes due to various impact magnitudes and then propagated under random fatigue loading. Since the causes of damage initiations are random in nature, the damage sizes can only be represented in terms of probabilistic distribution functions. In addition to the random initial damage size, applied load and debonding fracture toughness are also major uncertainty contributors in reliability assessment. For fleet risk management with maintenance planning, one should also include inspection reliability as represented by probability of detection (POD).

Traditionally, structural integrity (SI) was assessed based on a deterministic methodology with a set of safety factors to ensure SI. Due to the inherent uncertainties in many design and analysis parameters such as initial damage size, applied load, fracture toughness and crack growth rate, POD, etc., a probabilistic damage tolerance (DT) analysis with risk-based maintenance planning as depicted in Fig. 1 is necessary for fatigue life management and condition based maintenance of sandwich composite structures.

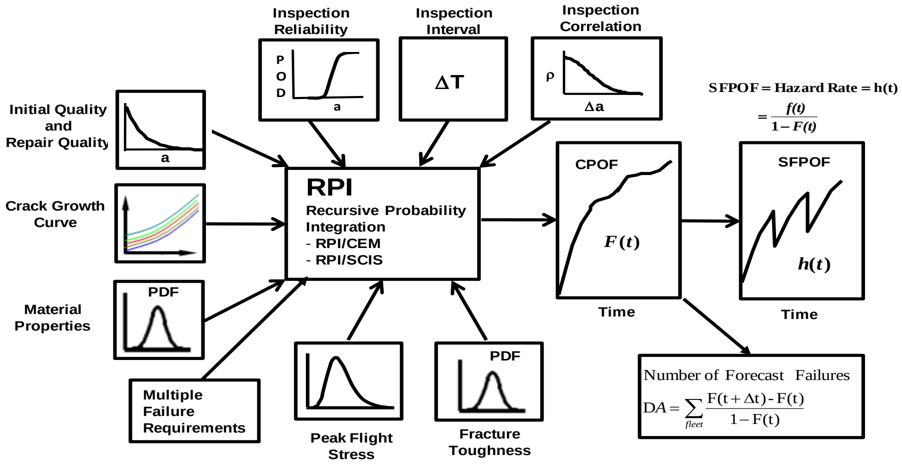

Fig. 1: Schematic of probabilistic damage tolerance analysis of aircraft components

Although these types of problems can normally be solved by Monte Carlo simulations (MCS), the computational efficiency using MCS to achieve a required accuracy is of great concern. To relieve the computational burden of traditional MCS methods and to further reduce the computational time for generating many sets of crack growth histories for various maintenance strategies, an efficient method is needed that combines the generality of MCS with the efficiency of analytical probabilistic methods. The core of the method used in this study is a recursive probabilistic integration (RPI) method [1, 2, 3, 4 and 5] that allows repeated use of baseline MCS-based crack growth histories for various maintenance plans. The fundamental concept of RPI is based on branching out the probable events after each maintenance action following an inspection where the POD is applied. The probability of occurrence of each branched event is then determined based on the probability of crack detection.

In this paper, we will present a study using the RPI probabilistic algorithm for DT and risk-based maintenance planning. A double cantilever beam made of sandwich composite materials was selected as the test configuration for simplicity. A closed form solution to determine the energy release rate based on the modified beam theory is used. The failure occurs when the stress intensity factor exceeds the debonding fracture toughness. In the following session, the RPI methodology will be described. A case study and RPI verification using sandwich DCB will be discussed in detail.

II. General Formulation Of Probabilistic Damage Tolerance Analysis With Maintenance Planning

To properly formulate probabilistic damage tolerance analysis with maintenance planning, we will define two types of the probability of failure and discuss the development of the probabilistic algorithm RPI (Recursive Probability Integration) in the following.

2.1 Definitions of Probability of Failure

There are several different measures of probability of failure due to various definitions. Those used in this study are defined below.

- The cumulative probability of failure (CPOF), *F*(t), is also the cumulative distribution function (CDF) of the fatigue life due to fracture.
- The hazard function *h*(t) is defined as the probability that a failure occurs in a unit time interval, given that a failure has not occurred prior to t, the beginning of the interval. The unit time can be a flight or a flight hour. The hazard function is also named single flight probability of failure (SFPOF) [6]. SFPOF is determined by Equation (1).

$$h(t) = f(t)/(1 - F(t)) \tag{1}$$

where *f*(t) is the probability density function (PDF) of the fatigue life.

2.2 Development of Recursive Probabilistic Integration (RPI) Method

Monte Carlo simulation (MCS) offers the most robust and reliable solution framework for general problems. The major issue is that MCS is time-consuming and unable to support timely decisions. For maintenance planning and risk monitoring, the computational issue is further amplified because the conventional approach requires a set of MCS for each different maintenance plan. As a result, numerous sets of MCSs with associated crack growth histories are required to search for the optimal solutions, such as best inspection intervals for minimum allowable risk by exploring the decision parameter space that consists of many possible combinations of inspection scheduling, techniques, and repair/replacement/retirement strategies.

To relieve the computational burden, an efficient method was developed which combined the generality of random simulations with the efficiency of analytical probabilistic methods. The core of the method is a recursive probabilistic integration (RPI) algorithm which allows repeated use of baseline simulation-based crack growth histories for various maintenance plans. The RPI for NDI applications is derived in Reference [5].

In traditional Monte Carlo simulations, random fatigue crack growth paths (or histories) are generated as a result of random crack growth behavior, random damage detections, random manufacturing, and random repair quality. A significant number of simulations are usually required for a desired accuracy. RPI systematically transforms the random pattern to a more logical and manageable probabilistic event tree for any MCS realizations. Using the event tree [7], state-of-the-art efficient reliability methods, including classical MPP-based and importance sampling methods [8], can be applied which results in a significant improvement on computational efficiency.

A detailed derivation of RPI method can be found in Reference [5]. As shown in the reference, a probabilistic event tree is used to derive the probabilistic algorithm RPI for the determination of cumulative probability of failure (CPOF) at time t. CPOF(t), which is equal to *F*(t) in Equation (1), is then derived as:

$$CPOF(t) = \frac{1}{n}\sum_{i=1}^{n}\sum_{N=0}^{m}\left[\begin{array}{l}(1-P_f^{cum,i}(N))*\left(P(\bigcap_{k=0}^{N-1}M_k^i)-P(\bigcap_{k=0}^{N}M_k^i)\right)*P_f(R_p^N)\\ +P\{\bigcap_{k=0}^{N}M_k^i\}*P_f^{\mathrm{int},i}(N,N+1)\end{array}\right] \quad (2)$$

where the subscript i refers to the i^{th} MCS simulation, n is the total number of simulations and m is the number of inspections conducted before time t for life assessment. All other terms in Equation (2) are defined in Reference [5]. Single flight probability of failure (SFPOF) at any given time in the interval [0, t] is then determined using Equation (1).

III. RPI Verification And Case Study

A damage tolerance and risk based fatigue life assessment of sandwich composites using the RPI method for risk prediction was selected for demonstration purposes. The sandwich composite is a double cantilever beam (DCB) as shown in Fig. 2. It is assumed that initial crack a_{init} exists at the upper face/core interface and the crack remains at or near this interface during crack propagation.

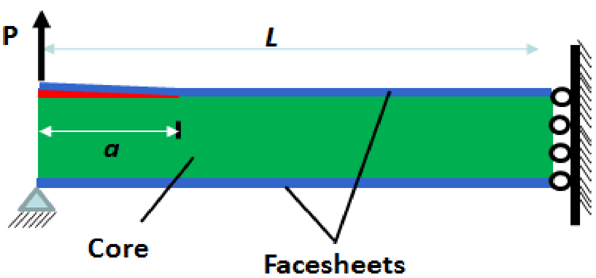

Fig. 2: Sandwich double cantilever beam

The Energy release rate G for Mode I fatigue crack growth is calculated using an analytical expression as shown in Equation (3) based on a data reduction method denoted as the “modified beam theory” (MBT) in the ASTM standard [9].

$$G = \frac{3P^2 C_c}{2b(a+|\Delta|)} \tag{3}$$

where b is the beam width and P is applied fatigue load and

$$C_c^{1/3} = m_c(a+|\Delta|) \tag{4}$$

The compliance $C_c^{1/3}$ is derived based on a DCB G/H100-thin specimen shown in Reference [10]. The associated parameters for Equations (3) and (4) are shown in Table 1.

Table 1: Parameters for G and $C^{1/3}$ Calculation

$\|\Delta\|$ (meter)	m_c	b (meter)
0.014	0.5909	0.025

The stress intensity factor is calculated by Equation (5) assuming a plain strain condition.

$$K_{\max} = \sqrt{G * E_c(1-\nu_c^2)} \tag{5}$$

where E_c (MPa) = 105 and ν_c =0.31. The crack growth rate is represented by the Paris equation as shown in Equation (6).

$$\frac{da}{dN} = C\Delta K^q \tag{6}$$

where

$$\Delta K = (1-R)K_{\max} \tag{7}$$

where R= K_{min}/K_{max}=0.1, C=3995 and q=7.59. The parameters C and m are based on Reference [11] for H100 core.

The maximum flight load P_{max} is modeled as a Gumbel distribution defined in Equation (8) where $h(P_{max})$ is the probability density function (PDF) of P_{max}.

$$h(P_{\max}) = \frac{1}{A} e^{-\frac{P_{\max}-B}{A}} e^{-e^{-\frac{P_{\max}-B}{A}}} \tag{8}$$

where A=1.0 and B=50.0.

For simplicity, the crack growth histories are generated by an equivalent load. Also, in this study, the inspections are based on NDI tools and the inspection measurements are assumed to be independent. As a consequence, $P\{[\bigcap_{k=0}^{N} M_k^i]\}$ in Equation (2) can be simplified to:

$$P\{[\bigcap_{k=0}^{N} M_k^i]\} = \prod_{k=0}^{N} (1-POD(a_k^i)) \tag{9}$$

The POD used in the study is characterized by Equation (10).

$$POD = \left\{1 + e^{\frac{-1.8[\ln(a)-\ln(a_{50})]}{\sigma}}\right\}^{-1} \qquad (10)$$

where a_{50} is the medium detectable crack length and σ is a scatter factor with a_{50} (meter) = 0.06 and σ=0.20.

The random variables considered are fracture toughness, initial crack a_{init}, and crack growth rate C. Their statistics and associated parameters are shown in Table 2. The maximum load in each flight is modeled as an independent random process. A constant inspection interval of 10,000 flights is assumed throughout the case study. The time increment for crack growth is set to be 100 flights as a previous study showed the crack growth histories converged with this time increment.

Table 2 : Statistics for Crack Growth Rate Parameter C and Fracture Toughness

Crack Growth Rate C			Fracture Toughness		
Mean	Standard Deviation	Distribution Type	Mean (MPa m$^{1/2}$)	Standard deviation (MPa m$^{1/2}$)	Distribution type
3995	120	Lognormal	0.21	0.021	Lognormal

A truncated Exponential distribution is selected for the initial crack size a_{init} as defined by Equation (11).

$$f_{a_{init}}(a_{init}) = \frac{e^{-a_{init}/a_m}}{a_m(1-e^{-a_{tr}/a_m})} \qquad (11)$$

where a_m (meter)= 0.02 and a_{tr} (meter) = 0.06.

The SFPOF predicted by RPI using 30 thousand random crack growth curves and that predicted by a traditional MCS analysis with 1.2 billion simulations are presented in Fig. 3. The SFPOF of RPI analysis can be obtained on desktop computer within three minutes. The 1.2 billion MCS run was conducted on 1280 computer cores with total 7600 CPU hours. The SFPOF of PRI agrees well with 1.2 billion MCS result. This study shows that the RPI method is 5 orders of magnitude more efficient than traditional MCS.

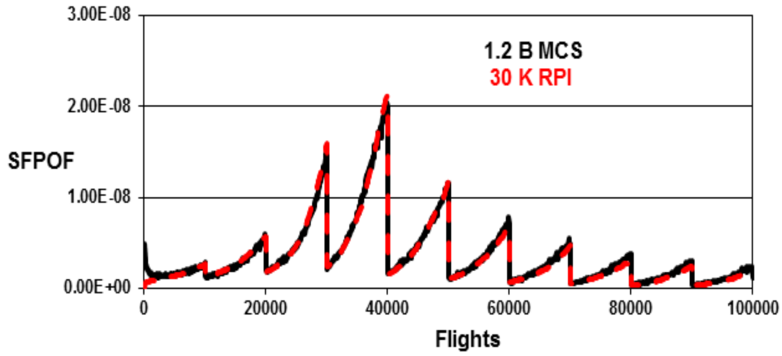

Fig. 3: Comparison of single flight probability of failure (SFPOF) predicted by Monte Carlo simulations and by RPI method

IV. Conclusion

A sandwich composite was probabilistically assessed for its fatigue life due to the disbond between the facesheet and the core. A methodology for probabilistic fleet risk management with maintenance planning of a debonded composite was demonstrated using a double cantilever beam (DCB) made of sandwich composite materials. An advanced and efficient probabilistic method, RPI (Recursive probability Integration), was used for risk calculations The RPI method combines random simulations with numerical integration that not only maintains the generality of random simulations but also takes advantage of the efficiency feature of numerical integration. Mode I fatigue crack growth analysis was conducted for damage tolerance assessment. Uncertainties in fracture toughness, crack growth rate, manufacturing and repair quality, were modeled probabilistically. Maximum flight loads was modeled as an independent random process. The study has demonstrated that RPI can be several orders of magnitude more efficient than that by traditional Monte Carlo simulations. Another unique feature of RPI method is that the number of potential random variables considered within the RPI framework is not limited.

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