Research Inventy: International Journal Of Engineering And Science
Vol.5, Issue 6 (June 2015), PP 24-28
Issn (e): 2278-4721, Issn (p):2319-6483, www.researchinventy.com

Time to Recruitment for a Single Grade Manpower System with Two Thresholds, Different Epochs for Inter-Decisions as an Order Statistics and Exits

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ABSTRACT: *In this paper, the problem of time to recruitment is studied using a univariate policy of recruitment involving two thresholds for a single grade manpower system with attrition generated by its policy decisions. Assuming that the policy decisions and exits occur at different epochs, a stochastic model is constructed and the variance of the time to recruitment is obtained when the inter-policy decision times and inter- exit times form an order statistics and an ordinary renewal process respectively. The analytical results are numerically illustrated with relevant findings by assuming specific distributions.*

Keywords: *Single grade manpower system; decision and exit epochs; order statistics; ordinary renewal process; univariate policy of recruitment with two thresholds and variance of the time to recruitment.*

I. INTRODUCTION

Attrition is a common phenomenon in many organizations. This leads to the depletion of manpower. Recruitment on every occasion of depletion of manpower is not advisable since every recruitment involves cost. Hence the cumulative depletion of manpower is permitted till it reaches a level, called the threshold. If the total loss of manpower exceeds this threshold, the activities in the organization will be affected and hence recruitment becomes necessary. In [1, 2, 3] the authors have discussed the manpower planning models using Markovian and renewal theoretic methods. In [4] the author has studied the problem of time to recruitment for a single grade manpower system and obtained the variance of the time to recruitment when the loss of manpower forms a sequence of independent and identically distributed random variables, the inter- decision times form a geometric process and the mandatory breakdown threshold for the cumulative loss of manpower is an exponential random variable by using univariate cum policy of recruitment. In [5] the author has studied the work in [4] using univariate and bivariate policies of recruitment both for the exponential mandatory threshold and for the one whose distribution has the SCBZ property. In [6] the author has initiated the study of the problem of time to recruitment for a single grade manpower system by incorporating alertness in the event of cumulative loss of manpower due to attrition crossing the threshold, by considering optional and mandatory thresholds for the cumulative loss of manpower in this manpower system. In [7] the authors have studied this problem when inter-decision times form an order statistics. In all the above cited work, it is assumed that attrition takes place instantaneously at decision epochs. This assumption is not realistic as the actual attrition will take place only at exit points which may or may not coincide with decision points. This aspect is taken into account for the first time in [8] and the variance of time to recruitment is obtained when the inter-decision times and exit times are independent and identically distributed exponential random variables using univariate policy for recruitment and Laplace transform in the analysis. In [9, 11, 12] the authors have extended their work in [8] when the inter-decision times form a sequence of exchangeable and constantly correlated exponential random variables, geometric process and order statistics respectively. Recently, in [13, 14, 15] the authors have studied the work in [8, 9, 11] respectively by considering optional and mandatory thresholds which is a variation from the work of [6] in the context of considering non-instantaneous exits at decision epochs. In the present paper, for a single grade manpower system, a mathematical model is constructed in which attrition due to policy decisions takes place at exit points and there are optional and mandatory thresholds as control limits for the cumulative loss of manpower. A univariate policy of recruitment based on shock model approach is used to determine the variance of time to recruitment when the system has different epochs for policy decisions and exits and the inter- exit times form an ordinary renewal process. The present paper studies the research work in [15] when the inter-policy decision times form an order statistics.

II. MODEL DESCRIPTION

Consider an organization taking decisions at random epochs in (0, ∞) and at every decision making epoch a random number of persons quit the organization. There is an associated loss of manpower if a person quits. It is assumed that the loss of manpower is linear and cumulative. Let X_i be the continuous random variable representing the amount of depletion of manpower (loss of man hours) caused at the i^{th} exit point and S_k be the total loss of manpower up to the first k exit points. It is assumed that X_i's are independent and identically distributed random variables with probability density function m(.), distribution function M(.) and mean $\frac{1}{\alpha}(\alpha > 0)$. Let U_k be the continuous random variable representing the time between the (k-1)th and kth policy decisions. It is assumed that U_k's are independent and identically distributed random variables with distribution F(.) and probability density function $f(.)$.Let $F_{a(j)}(.)$ and $f_{a(j)}(.)$ be the distribution and the probability density function of the j^{th} order statistic ($j = 1,2,...,r$) selected from the sample of size r from the population $\{U_k\}_{k=1}^{\infty}$. Let W_i be the continuous random variable representing the time between the (i-1)th and ith exit points. It is assumed that W_i's are independent and identically distributed random variables with probability density function g(.), probability distribution function G(.). Let $N_e(t)$ be the number of exit points lying in (0, t]. Let Y be the optional threshold level and Z the mandatory threshold level (Y< Z) for the cumulative depletion of manpower in the organization with probability density function h(.) and distribution function H(.). Let p be the probability that the organization is not going for recruitment when optional threshold is exceeded by the cumulative loss of manpower. Let q be the probability that every policy decision has exit of personnel. As q=0 corresponds to the case where exits are impossible, it is assumed that $q \neq 0$. Let T be the random variable denoting the time to recruitment with probability distribution function L(.), density function l (.), mean E(T) and variance V(T). Let $\bar{a}(.)$ be the Laplace transform of a(.). The univariate CUM policy of recruitment employed in this paper is stated as follows:

Recruitment is done whenever the cumulative loss of manpower in the organization exceeds the mandatory threshold. The organization may or may not go for recruitment if the cumulative loss of manpower exceeds the optional threshold.

III. MAIN RESULT

As in [15], we get

$$P(T > t) = \sum_{k=0}^{\infty} P[N_e(t) = k]\, P(S_k \le Y) + p \sum_{k=0}^{\infty} P[N_e(t) = k]\, P(S_k > Y)\, P(S_k \le Z) \quad (1)$$

$$L(t) = \bar{a}\sum_{k=1}^{\infty} G_k(t)\, a^{k-1} + p\left\{ \bar{b}\sum_{k=1}^{\infty} G_k(t)\, b^{k-1} - \overline{ab}\sum_{k=1}^{\infty} G_k(t)\,(ab)^{k-1} \right\} \quad (2)$$

and

$$\bar{l}(s) = \frac{\bar{a}\, q\, \bar{f}(s)}{1 + (\bar{a}\, q - 1)\bar{f}(s)} + p\left\{ \frac{\bar{b}\, q\, \bar{f}(s)}{1 + (\bar{b}\, q - 1)\bar{f}(s)} - \frac{\overline{ab}\, q\, \bar{f}(s)}{1 + (\overline{ab}\, q - 1)\bar{f}(s)} \right\} \quad (3)$$

where

$$\bar{a} = 1 - a, \bar{b} = 1 - b, \overline{ab} = 1 - ab \quad (4)$$

Let $U_1, U_2, ..., U_r$ be arranged in increasing order of magnitude so that we have the sequence as $U_{(1)}, U_{(2)}, ..., U_{(r)}$, here $r = 1, 2, ...,\ j = 1, 2, ..., r$ are called the order statistics and with $U_{(1)}$ as the first order statistic and $U_{(r)}$ the rth order statistic and the random variables $U_{(1)}, U_{(2)}, ..., U_{(r)}$ are not independent. The probability density function of the j^{th} order statistic is given by

$$f_{a(j)}(t) = j \binom{r}{j} [F(t)]^{j-1} f(t)[1-F(t)]^{r-j}, \tag{5}$$

where it is assumed that $F(t) = 1 - e^{-\lambda t}$.

Case (i): $f(t) = f_{a(1)}(t)$

In this case, from (3) and (5), we get

$$\bar{l}(s) = \frac{\bar{a}\, q\, \bar{f}_{a(1)}(s)}{1+(\bar{a}\, q-1)\,\bar{f}_{a(1)}(s)} + p\left\{\frac{\bar{b}\, q\, \bar{f}_{a(1)}(s)}{1+(\bar{b}\, q-1)\,\bar{f}_{a(1)}(s)} - \frac{\overline{ab}\, q\, \bar{f}_{a(1)}(s)}{1+(\overline{ab}\, q-1)\,\bar{f}_{a(1)}(s)}\right\}, \tag{6}$$

where

$$\bar{f}_{a(1)}(s) = \frac{r\lambda}{r\lambda + s} \tag{7}$$

Substituting (7) in (6) and on simplification, we get

$$\bar{l}(s) = \frac{\bar{a}\, q\, r\lambda}{(r\lambda+s)+(\bar{a}\, q-1)r\lambda} + p\left\{\frac{\bar{b}\, qr\lambda}{(r\lambda+s)+(\bar{b}\, q-1)r\lambda} - \frac{\overline{ab}\, qr\lambda}{(r\lambda+s)+(\overline{ab}\, q-1)r\lambda}\right\} \tag{8}$$

$$E(T) = \frac{d}{ds}\left[\bar{l}(s)\right]_{s=0} = \frac{1}{\bar{a}qr\lambda} + p\left\{\frac{1}{\bar{b}qr\lambda} - \frac{1}{\overline{ab}qr\lambda}\right\} \tag{9}$$

and

$$V(T) = \frac{d^2}{ds^2}\left[\bar{l}(s)\right]_{s=0} - [E(T)]^2 = \frac{2}{(qr\lambda)^2}\left\{\frac{1}{(\bar{a})^2} + \frac{p}{(\bar{b})^2} - \frac{p}{(\overline{ab})^2}\right\} - \frac{1}{(qr\lambda)^2}\left\{\frac{1}{\bar{a}} + \frac{p}{\bar{b}} - \frac{p}{\overline{ab}}\right\}^2 \tag{10}$$

where $\bar{a}$, $\bar{b}$, $\overline{ab}$ are given by (4).

Case (ii): $f(t) = f_{a(r)}(t)$.

In this case, from (3) and (5), we get

$$\bar{l}(s) = \frac{\bar{a}\, q\, \bar{f}_{a(r)}(s)}{1+(\bar{a}\, q-1)\,\bar{f}_{a(r)}(s)} + p\left\{\frac{\bar{b}\, q\, \bar{f}_{a(r)}(s)}{1+(\bar{b}\, q-1)\,\bar{f}_{a(r)}(s)} - \frac{\overline{ab}\, q\, \bar{f}_{a(r)}(s)}{1+(\overline{ab}\, q-1)\,\bar{f}_{a(r)}(s)}\right\} \tag{11}$$

where

$$\bar{f}_{a(r)}(s) = \frac{r!\,\lambda^r}{(\lambda+s)(2\lambda+s)\ldots(r\lambda+s)} \tag{12}$$

Substituting (12) in (11) and on simplification, we get

$$\bar{l}(s) = \frac{\bar{a}\, q\, r!\,\lambda^r}{C+(\bar{a}\, q-1)\, r!\,\lambda^r} + p\left\{\frac{\bar{b}\, q\, r!\,\lambda^r}{C+(\bar{b}\, q-1)\, r!\,\lambda^r} - \frac{\overline{ab}\, q\, r!\,\lambda^r}{C+(\overline{ab}\, q-1)\, r!\,\lambda^r}\right\} \tag{13}$$

where

$$C = (\lambda+s)(2\lambda+s)\ldots(r\lambda+s) \text{ and } [C]_{s=0} = r!\,\lambda^r \tag{14}$$

Proceeding as in case(i), we get

$$E(T) = \frac{1}{q\lambda}\sum_{j=1}^{r}\frac{1}{j}\ \left\{\frac{1}{\bar{a}} + \frac{p}{\bar{b}} - \frac{p}{\overline{ab}}\right\} \tag{15}$$

and

$$V(T) = \frac{1}{(q\lambda)^2}\left(\sum_{j=1}^{r}\frac{1}{j}\right)^2\left\{\frac{2-\bar{a}q}{(\bar{a})^2} + \frac{p(2-\bar{b}q)}{(\bar{b})^2} - \frac{p(2-\overline{ab}q)}{(\overline{ab})^2}\right\} + \frac{1}{q\lambda^2}\sum_{j=1}^{r}\frac{1}{j^2}\left\{\frac{1}{\bar{a}} + \frac{p}{\bar{b}} - \frac{p}{\overline{ab}}\right\}$$
$$- \frac{1}{(q\lambda)^2}\left(\sum_{j=1}^{r}\frac{1}{j}\right)^2\left\{\frac{1}{\bar{a}} + \frac{p}{\bar{b}} - \frac{p}{\overline{ab}}\right\}^2 \tag{16}$$

where $\bar{a}\,,\bar{b}\,,\overline{ab}$ are given by (4).

Equations (9), (10) and (15), (16) give the mean and variance on the time to recruitment for Cases (i) and (ii) respectively.

Remark:

Computation of $V(T)$ for extended exponential and SCBZ Property possessing thresholds is similar as their distribution will have just additional exponential terms.

Note:

(i)When p=0 and r =1, our results in Case (i) agree with the results in [8] for the manpower system having only one threshold which is the mandatory threshold.

(ii)When p=0 and $r \neq 1$ our results in Cases (i) and (ii) agree with the results in [12] for the manpower system having only one threshold which is the mandatory threshold.

(iii)When q=1and r =1, our results in Case (i) agree with the results in [6] for the manpower system having certain instantaneous exits in the decision epochs.

(iv)When r =1, $p \neq 0, q \neq 1$ our results in Case (i) agree with the results in [13] for the manpower system having inter-decision times as an ordinary renewal process.

IV. NUMERICAL ILLUSTRATION

The mean and variance of time to recruitment is numerically illustrated by varying one parameter and keeping other parameters fixed. The effect of the nodal parameters $\alpha\,,\lambda\,,r$ on the performance measures is shown in the following table. In the computations, it is assumed that $\theta_1 = 0.06,\ \theta_2 = 0.09, q = 0.2\,, p = 0.5$.

Table: Effect of nodal parameters on E(T) and V(T)

α	λ	r	Case(i)		Case(ii)	
			E(T)	V(T)	E(T)	V(T)
0.2	1	5	4.8795	19.5924	55.6992	$2.4614\text{x}10^3$
0.2	1	6	4.0662	13.6058	59.7491	$2.8277\text{x}10^3$
0.2	1	7	3.4853	9.9961	60.7982	$2.9271\text{x}10^3$
0.2	1	8	3.0497	7.6533	63.8479	$3.2247\text{x}10^3$
0.2	2	5	2.4397	4.8981	27.8496	615.3589
0.2	3	5	1.6265	2.1769	18.5664	273.4928
0.2	4	5	1.2199	1.2245	13.9248	153.8397
0.2	5	5	0.9759	0.7837	11.1398	98.4574
0.3	1	5	6.7738	37.1331	77.3230	$4.7115\text{x}10^3$
0.4	1	5	8.6656	60.2258	98.9184	$7.6851\text{x}10^3$
0.5	1	5	10.5564	88.8727	120.5011	$1.1382\text{x}10^4$
0.6	1	5	12.4465	123.0743	142.0773	$1.5803\text{x}10^4$

V. FINDINGS

From the above tables, the following observations are presented which agree with reality.

1. When α increases and keeping all the other parameters fixed, the average loss of manpower increases. Therefore the mean and variance of time to recruitment increase.
2. As λ increases, on the average, the inter-decision time decreases and consequently the mean and variance of time to recruitment decrease when the other parameters are fixed.
3. When r increases and keeping all the other parameters fixed, the mean and variance of the time to recruitment decrease in Case (i) and increase in Case (ii).

VI. CONCLUSION

The models discussed in this paper are found to be more realistic and new in the context of considering (i) separate points (exit points) on the time axis for attrition, thereby removing a severe limitation on instantaneous attrition at decision epochs (ii) associating a probability for any decision to have exit points and (iii) provision of optional and mandatory thresholds. From the organization's point of view, our models are more suitable than the corresponding models with instantaneous attrition at decision epochs, as the provision of exit points at which attrition actually takes place, postpone the time to recruitment.

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