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Common Fixed Theorems Using Random Implicit Iterative Schemes

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ABSTRACT : *The aim of this paper is to prove two convergence theorems of two random implicit iterative schemes to a common random fixed point in Banach spaces.*

MATHEMATICS SUBJECT CLASSIFICATION: *47H10, 54H25*

KEY WORDS*: Contractive Mappings, Implicit Iterative Schemes, Measurable Mappings, Separable Banach Spaces.*

I. INTRODUCTION AND PRELIMINARIES

Random approximations and random fixed point theorems are stochastic generalizations of classical approximations and fixed point theorems. Random fixed point theory was initiated by the Prague school of probabilities in the works of Hans [12] and Spacek [2]. Fixed point iterative schemes for nonlinear operators on Banach and Hilbert spaces were studied and improved by many authors in recent times. The development of random fixed point iterative schemes was initiated by Choudhary in [4] where random Ishikawa iterative scheme was defined and its strong convergence to a random fixed point in Hilbert spaces was discussed. After that several authors [1, 9, 13, 14, 15, 19] have worked on random fixed point iterations to obtain fixed points in deterministic operator theory. Then Chugh et al.[16] defined and proved the convergence of random SP iterative scheme

The following iterative schemes are now well known:

Random Mann iterative scheme [3]:

$$x_{n+1}(w) = (1-\alpha_n)x_n(w) + \alpha_n T(w, x_n(w)), \text{ for } n > 0, \quad w \in \Omega, \tag{1.1}$$

where $0 \le \alpha_n \le 1$ and $x_0 : \Omega \to F$ is an arbitrary measurable mapping.

Random Ishikawa iterative scheme [4]:

$$x_{n+1}(w) = (1-\alpha_n)x_n(w) + \alpha_n T(w, y_n(w)),$$

$$y_n(w) = (1-\beta_n)x_n(w) + \beta_n T(w, x_n(w)), \text{ for } n > 0, \quad w \in \Omega, \tag{1.2}$$

where $0 \le \alpha_n, \beta_n \le 1$ and $x_0 : \Omega \to F$ is an arbitrary measurable mapping.

Random SP iterative scheme [16]:

$$\begin{aligned} x_{n+1}(w) &= (1-\alpha_n)y_n(w) + \alpha_n T(w, y_n(w)), \\ y_n(w) &= (1-\beta_n)z_n(w) + \beta_n T(w, z_n(w)), \\ z_n(w) &= (1-\gamma_n)x_n(w) + \gamma_n T(w, x_n(w)) \text{ for } n > 0, \quad w \in \Omega, \end{aligned} \tag{1.3}$$

where $\{\alpha_n\}$, $\{\beta_n\}$ and $\{\gamma_n\}$ are sequences of positive numbers in [0,1] and $x_0 : \Omega \to F$ is an arbitrary measurable mapping.

In 2001, Xu and Ori [7], introduced the following implicit iterative scheme for a finite family of nonexpansive mappings $\{T_1, T_2, \ldots, T_N\}$ from K to K, where K is a nonempty closed convex subset of a Hilbert space E. Let $\{\alpha_n\}$ be a real sequence in (0, 1) and an initial point $x_1 \in C$,

$$
\begin{aligned}
x_1 &= \alpha_1 x_0 + (1-\alpha_1) T_1 x_1, \\
x_2 &= \alpha_2 x_1 + (1-\alpha_2) T_2 x_2, \\
&\ldots \\
x_N &= \alpha_N x_{N-1} + (1-\alpha_N) T_N x_N, \\
x_{N+1} &= \alpha_{N+1} x_N + (1-\alpha_{N+1}) T_1 x_{N+1}, \\
&\ldots
\end{aligned}
\tag{1.4}
$$

It can also be written as

$x_n = \alpha_n x_{n-1} + (1-\alpha_n) T_n x_n$, $\forall\, n \geq 1$, where $T_n = T_{n (\mathrm{mod}\, N)}$ (mod N takes the values in the set {1, 2, … , N}). They proved the weak convergence of the sequence $\{x_n\}$ defined by (1.4) to a common fixed point

$$p \in F = \bigcap_{i=1}^{N} F(T_i)$$

Corresponding to this, I. Beg and B. S. Thakur [8] defined general composite random implicit iterative scheme as follows:

Let $S_i, T_i : \Omega \times C \to C$, i = 1, 2, …, N be operators on a nonempty convex subset *C* of a separable Banach space *X*. Then the sequence $\{x_n\}$ generated by random implicit scheme associated with S_i or T_i is defined as follows:

Let $x_0 : \Omega \to C$ be any given measurable mapping.

$$
\begin{aligned}
x_1(t) &= (1-\alpha_1) x_0(t) + \alpha_1 S_1(t, x_1(t)) \\
x_2(t) &= (1-\alpha_2) x_1(t) + \alpha_2 S_2(t, x_2(t)) \\
&. \\
&. \\
&. \\
x_N(t) &= (1-\alpha_N) x_{N-1}(t) + \alpha_N S_N(t, x_N(t)) \\
x_{N+1}(t) &= (1-\alpha_{N+1}) x_N(t) + \alpha_{N+1} S_1(t, x_{N+1}(t)) \\
&. \\
&. \\
&. \\
x_{2N}(t) &= (1-\alpha_{2N}) x_{2N-1}(t) + \alpha_{2N} S_N(t, x_{2N}(t)) \\
&. \\
&.
\end{aligned}
\tag{1.5}
$$

or

$$x_1(t) = (1-\alpha_1)\,x_0\left(t\right) + \alpha_1 T_1(t, x_1(t))$$
$$x_2(t) = (1-\alpha_2)\,x_1\left(t\right) + \alpha_2 T_2(t, x_2(t))$$
.
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.
$$x_N(t) = (1-\alpha_N)\,x_{N-1}\left(t\right) + \alpha_N T_N(t, x_N(t))$$
$$x_{N+1}(t) = (1-\alpha_{N+1})\,x_N\left(t\right) + \alpha_{N+1} T_1(t, x_{N+1}(t)) \tag{1.6}$$
.
.
.
$$x_{2N}(t) = (1-\alpha_{2N})\,x_{2N-1}\left(t\right) + \alpha_{2N} T_N(t, x_{2N}(t))$$
.
.

where $\{\alpha_n\} \subseteq [0,1]$.

Definition Let $\{T_1, T_2, \ldots, T_N\}$ be a family of random asymptotically nonexpansive operators from $\Omega \times K \to K$, where K is a closed, convex subset of a separable Banach space E. Let $F = \bigcap_{i=1}^{N} RF(T_i) \neq \phi$, where $RF(T_i)$ is the set of all random fixed points of a random operator T_i for each i ∈ {1, 2,…, N}. Let $\xi_0 : \Omega \to K$ be any fixed measurable mapping and $\{\alpha_n\} \subset [0,1]$, then modified implicit random iterative scheme associated with S_i is defined as follows:

$$x_1(t) = (1-\alpha_1)\,x_0\left(t\right) + \alpha_1 S_1(t, x_1(t))$$
$$x_2(t) = (1-\alpha_2)\,x_1\left(t\right) + \alpha_2 S_2(t, x_2(t))$$
.
.
.
$$x_N(t) = (1-\alpha_N)\,x_{N-1}\left(t\right) + \alpha_N S_N(t, x_N(t))$$
$$x_{N+1}(t) = (1-\alpha_{N+1})\,x_N\left(t\right) + \alpha_{N+1} S_1^{\,2}(t, x_{N+1}(t))$$
.
.
.
$$x_{2N}(t) = (1-\alpha_{2N})\,x_{2N-1}\left(t\right) + \alpha_{2N} S_N^{\,2}(t, x_{2N}(t))$$
$$x_{2N+1}(t) = (1-\alpha_{2N+1})\,x_{2N}\left(t\right) + \alpha_{2N+1} S_1^{\,3}(t, x_{2N+1}(t)), \tag{1.7}$$
.
.

Similarly, modified implicit random iterative scheme associated with T_i, is defined as follows:

$x_1(t) = (1-\alpha_1)\, x_0(t) + \alpha_1 T_1(t, x_1(t))$

$x_2(t) = (1-\alpha_2)\, x_1(t) + \alpha_2 T_2(t, x_2(t))$

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$x_N(t) = (1-\alpha_N)\, x_{N-1}(t) + \alpha_N T_N(t, x_N(t))$

$x_{N+1}(t) = (1-\alpha_{N+1})\, x_N(t) + \alpha_{N+1} T_1^{\,2}(t, x_{N+1}(t))$

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$$x_{2N}(t) = (1-\alpha_{2N})\, x_{2N-1}(t) + \alpha_{2N} T_N^{\,2}(t, x_{2N}(t)) \tag{1.8}$$

$x_{2N+1}(t) = (1-\alpha_{2N+1})\, x_{2N}(t) + \alpha_{2N+1} T_1^{\,3}(t, x_{2N+1}(t)),$

.

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In this paper, we prove the convergence of two random implicit iterative schemes to a random common fixed point . Our result is generalization of the results in [16] and some other known results [10, 20] in the literature of fixed point theory. Firstly we give some definitions.

Through this paper, $(\Omega, \sum)$ denotes a measurable space and X denotes a real Banach space. For any function $T : \Omega \times X \to X$ we denote the n-th iterate $T(t, T(t, \ldots, T(t, x)))$ of T by $T^n(t, x)$. The letter I denotes the random mapping $I : \Omega \times X \to X$ defined by $I(\omega, x) = x$ and $T^0 = I$.

Definition 1.1 Let C be a nonempty subset of a separable Banach space X and $T : \Omega \times C \to C$ be a random operator. Then T is said to be an asymptotically nonexpansive random operator if there exists a sequence of measurable functions $r_n : \Omega \to [1, \infty)$ with $\lim_{n \to \infty} r_n(t) = 1$ such that

$$\left\| T^n(t, x) - T^n(t, y) \right\| \leq r_n(t) \left\| x - y \right\|$$

for all $x, y \in C, n \in N$ and for each $t \in \Omega$.

Definition 1.2 A mapping $f : \Omega \to C$ is called measurable if $f^{-1}(B \cap C) \in \sum$ for every Boral subset *B* of *X*.

Definition 1.3 A function $F : \Omega \times C \to C$ is called a random operator if $F(., x) : \Omega \to C$ is measurable for every $x \in C$.

Definition 1.4 A measurable mapping $g : \Omega \to C$ is said to be random fixed point of the random operator $F : \Omega \times C \to C$, if $F(w, g(w)) = g(w)$ for all $w \in \Omega$.

Definition 1.5 A random operator $F : \Omega \times C \to C$ is said to be continuous if, for fixed $w \in \Omega$, $F(w, .) : C \to C$ is continuous.

Now, we prove our main results.

II. MAIN RESULTS

Theorem 2.1 Let X be a separable Banach space and C be a nonempty, closed and convex subset of X. Let $S_i, T_j : \Omega \times C \to C$, i, j ∈ {1, 2,…, N} be random operators defined on C such that at least one of the following conditions hold for all $x, y \in C$ and $t \in \Omega$:

(i) $\|S_i(t,x) - T_j(t,y)\| \le$

$$a\|x-y\| + b\left[\|x - S_i(t,x)\| + \|y - T_j(t,y)\|\right] + c\left[\|x - T_j(t,y)\| + \|y - S_i(t,x)\|\right],$$

$a > 0,\ b \ge 0,\ c \ge 0, 1 - b - c > 0.$

(ii) $\|S_i(t,x) - T_j(t,y)\| \le$

$$q\max\left\{\|x-y\|, \|x - S_i(t,x)\| + \|y - T_j(t,y)\|, \|x - T_j(t,y)\| + \|y - S_i(t,x)\|\right\},$$

$0 < q < 1.$

(iii) $\|S_i(t,x) - T_j(t,y)\| \le$

$$\alpha\max\left\{\beta\|x-y\|, \|x - S_i(t,x)\|, \|y - T_j(t,y)\|, \|x - T_j(t,y)\|, \|y - S_i(t,x)\|\right\},$$

$\alpha, \beta \ge 0,\ 0 \le \alpha < 1.$

(iv) $\|S_i(t,x) - T_j(t,y)\| \le$

$$q\max\left\{\|x-y\|, \|x - S_i(t,x)\|, \|y - T_j(t,y)\|, \|x - T_j(t,y)\|, \|y - S_i(t,x)\|\right\},$$

$0 < q < 1.$

If the random implicit iterative scheme associated with (1.5) or (1.6), satisfying (i)-(iii) converges, then it converges to a common random fixed point of S_i and T_j. Further, if (iv) holds, then this common fixed point will be unique.

Proof. Let us assume that the sequence $\{x_n\}$ defined by (1.5) has a pointwise limit, that is, $\lim_{n\to\infty} x_n(t) = u(t)$, for all $t \in \Omega$. As X is a separable Banach space, the mapping $x(t) = A(t, f(t))$ is measurable mapping [6] for any random operator $A : \Omega \times C \to X$ and any measurable mapping $f : \Omega \to C$.

Now, the sequence $\{x_n\}$ constructed by the random implicit iterative schemes (1.5) and (1.6) is a sequence of measurable mappings as $x(t)$ is measurable and C is convex. Therefore, $x : \Omega \to C$ is also measurable, being limit of a sequence measurable mappings.

First of all we assume that $S_i(t, x(t)) = x(t)$ for $x(t) \in C$. Then after putting $x(t) = y(t) = u(t)$ into any of the inequalities $(i) - (iv)$, it is easy to see that $T_j(t, u(t)) = u(t)$. In a similar manner $T_j(t, u(t)) = u(t)$ implies $S_i(t, u(t)) = u(t)$

Suppose the sequence $\{x_n\}$ generated by implicit iterative scheme associated with S_i converges to u, that is, $\lim_{n\to\infty} x_n(t) = u(t)$. Then, from (1.5), we have

$x_{n+1}(t) = (1 - \alpha_{n+1})x_n(t) + \alpha_{n+1}S_i(t, x_{n+1}(t))$. Since $\lim_{n\to\infty} x_n(t) = u(t)$, so we have $\|x_{n+1}(t) - x_n(t)\| \to 0$. Using it, we obtain $\|x_n(t) - S_i(t, x_{n+1}(t))\| \to 0$. From which, it follows that $\|u(t) - S_i(t, x_{n+1}(t))\| \to 0$

If S_i, T_j satisfy (i), then

$$\left\|S_i\left(t, x_n(t)\right) - T_j\left(t, u(t)\right)\right\| \le a\left\|x_n(t) - u(t)\right\| + b\left[\left\|x_n(t) - S_i\left(t, x_n(t)\right)\right\| + \left\|u(t) - T_j(t, u(t))\right\|\right]$$
$$+ c\left[\left\|x_n(t) - T_j\left(t, u(t)\right)\right\| + \left\|u(t) - S_i\left(t, x_n(t)\right)\right\|\right]. \tag{2.1}$$

If S_i, T_j satisfy (ii), then

$$\left\|S_i\left(t, x_n(t)\right) - T_j\left(t, u(t)\right)\right\| \le q \max\left\{\begin{array}{l}\left\|x_n(t) - u(t)\right\|, \left\|x_n(t) - S_i\left(t, x_n(t)\right)\right\| + \left\|u(t) - T_j\left(t, u(t)\right)\right\|, \\ \left\|x_n(t) - T_j\left(t, u(t)\right)\right\| + \left\|u(t) - S_i\left(t, x_n(t)\right)\right\|\end{array}\right\} \tag{2.2}$$

Also, if S_i, T_j satisfy (iii), then

$$\left\|S_i\left(t, x_n(t)\right) - T_j\left(t, u(t)\right)\right\| \le \alpha \max\left\{\begin{array}{l}\alpha_n\left\|x_n(t) - u(t)\right\|, \left\|x_n(t) - S_i\left(t, x_n(t)\right)\right\|, \left\|u(t) - T_j\left(t, u(t)\right)\right\|, \\ \left\|x_n(t) - T_j\left(t, u(t)\right)\right\|, \left\|u(t) - S_i\left(t, x_n(t)\right)\right\|\end{array}\right\} \tag{2.3}$$

Further, if T_i, S_j satisfy (iv), then clearly they will satisfy (ii).
On applying the limit $n \to \infty$, in (2.1), (2.2), (2.3), we get

$$\left\|u(t) - T_j\left(t, u(t)\right)\right\| \le \lambda\left\|u(t) - T_j\left(t, u(t)\right)\right\|, \tag{2.4}$$

where $\lambda = \max\{b + c, q, \alpha\} < 1$. Then $T_j\left(t, u(t)\right) = u(t)$. Similarly, we can show that $S_i\left(t, u(t)\right) = u(t)$.

To prove the uniqueness of $u(t)$ in the case (iv), let us assume that $v(t)$ be common fixed point of S and T other than $u(t)$, then using (iv), we have

$$\begin{aligned}\left\|u(t) - v(t)\right\| &= \left\|S_i\left(t, u(t)\right) - T_j\left(t, v(t)\right)\right\| \\ &\le q \max\left\{\begin{array}{l}\left\|u(t) - v(t)\right\|, \left\|u(t) - S_i\left(t, u(t)\right)\right\|, \\ \left\|v(t) - T_j\left(t, v(t)\right)\right\|, \left\|u(t) - T_j\left(t, v(t)\right)\right\|, \left\|v(t) - S_i\left(t, u(t)\right)\right\|\end{array}\right\} \\ &\le q\left\|u(t) - v(t)\right\|,\end{aligned}$$

which further yields $(1 - q)\left\|u(t) - v(t)\right\| \le 0$.

But $0 < q < 1$, hence $\left\|u(t) - v(t)\right\| < 0$, which is a contradiction as norm is always nonnegative, therefore $u(t) = v(t)$ always.

Theorem 2.2 Let X be a separable Banach space and C be a nonempty, closed and convex subset of X. Let $S_i, T_j : \Omega \times C \to C$, $i, j \in \{1, 2, \ldots, N\}$ be random operators defined on C such that at least one of the following conditions hold for all $x, y \in C$ and $t \in \Omega$:

(i) $\left\|S_i(t,x)-T_j(t,y)\right\| \le$

$$a\left\|x-y\right\|+b\left[\left\|x-S_i(t,x)\right\|+\left\|y-T_j(t,y)\right\|\right]+c\left[\left\|x-T_j(t,y)\right\|+\left\|y-S_i(t,x)\right\|\right],$$

$a>0,\ b\ge 0,\ c\ge 0, 1-b-c>0.$

(ii) $\left\|S_i(t,x)-T_j(t,y)\right\| \le$

$$q\max\left\{\left\|x-y\right\|,\left\|x-S_i(t,x)\right\|+\left\|y-T_j(t,y)\right\|,\left\|x-T_j(t,y)\right\|+\left\|y-S_i(t,x)\right\|\right\},$$

$0<q<1.$

(iii) $\left\|S_i(t,x)-T_j(t,y)\right\| \le$

$$\alpha\max\left\{\beta\left\|x-y\right\|,\left\|x-S_i(t,x)\right\|,\left\|y-T_j(t,y)\right\|,\left\|x-T_j(t,y)\right\|,\left\|y-S_i(t,x)\right\|\right\},$$

$\alpha,\beta\ge 0,\ 0\le\alpha<1.$

(iv) $\left\|S_i(t,x)-T_j(t,y)\right\| \le$

$$q\max\left\{\left\|x-y\right\|,\left\|x-S_i(t,x)\right\|,\left\|y-T_j(t,y)\right\|,\left\|x-T_j(t,y)\right\|,\left\|y-S_i(t,x)\right\|\right\},$$

$0<q<1.$

If the random implicit iterative scheme associated with (1.7) or (1.8) satisfying (i)-(iii) converges, then it converges to a common random fixed point of S_i and T_j. Further, if (iv) holds, then this common fixed point will be unique.

Proof. Let us assume that the sequence $\{x_n\}$ defined by (1.7) has a pointwise limit, that is, $\lim_{n\to\infty} x_n(t)=u(t)$, for all $t\in\Omega$. As X is a separable Banach space, the mapping $x(t)=A(t,f(t))$ is measurable mapping [6] for any random operator $A:\Omega\times C\to X$ and any measurable mapping $f:\Omega\to C$.

Now, the sequence $\{x_n\}$ constructed by the random implicit iterative scheme (1.7) and (1.8) is a sequence of measurable mappings as $x(t)$ is measurable and C is convex. Therefore, $x:\Omega\to C$ is also measurable, being limit of a sequence measurable mappings.

First of all we assume that $S_i(t,x(t))=x(t)$ for $x(t)\in C$. Then after putting $x(t)=y(t)=u(t)$ into any of the inequalities $(i)-(iv)$, it is easy to see that $T_j(t,u(t))=u(t)$. In a similar manner $T_j(t,u(t))=u(t)$ implies $S_i(t,u(t))=u(t)$

Suppose the sequence $\{x_n\}$ generated by implicit iterative scheme associated with S_i converges to u, that is, $\lim_{n\to\infty} x_n(t)=u(t)$. Then, from (1.7) we have

$x_{n+1}(t)=(1-\alpha_{n+1})x_n(t)+\alpha_{n+1}S_i^j(t,x_{n+1}(t))$. Since $\lim_{n\to\infty} x_n(t)=u(t)$, so we have $\left\|x_{n+1}(t)-x_n(t)\right\|\to 0$. Using it, we obtain $\left\|x_n(t)-S_i(t,x_{n+1}(t))\right\|\to 0$. From which, it follows that $\left\|u(t)-S_i^j(t,x_{n+1}(t))\right\|\to 0$

If S_i, T_j satisfy (i), then

$$\left\|S_i(t,x_n(t))-T_j(t,u(t))\right\|\le a\left\|x_n(t)-u(t)\right\|+b\left[\left\|x_n(t)-S_i(t,x_n(t))\right\|+\left\|u(t)-T_j(t,u(t))\right\|\right]$$
$$+c\left[\left\|x_n(t)-T_j(t,u(t))\right\|+\left\|u(t)-S_i(t,x_n(t))\right\|\right]. \tag{2.5}$$

If S_i,T_j satisfy (ii), then

$$\left\|S_i\left(t, x_n(t)\right) - T_j\left(t, u(t)\right)\right\| \le q \max\left\{\begin{array}{l} \left\|x_n(t) - u(t)\right\|, \left\|x_n(t) - S_i\left(t, x_n(t)\right)\right\| + \left\|u(t) - T_j\left(t, u(t)\right)\right\|, \\ \left\|x_n(t) - T_j\left(t, u(t)\right)\right\| + \left\|u(t) - S_i\left(t, x_n(t)\right)\right\| \end{array}\right\} \tag{2.6}$$

Also, if S_i, T_j satisfy (iii), then

$$\left\|S_i\left(t, x_n(t)\right) - T_j\left(t, u(t)\right)\right\| \le \alpha \max\left\{\begin{array}{l} \alpha_n \left\|x_n(t) - u(t)\right\|, \left\|x_n(t) - S_i\left(t, x_n(t)\right)\right\|, \left\|u(t) - T_j\left(t, u(t)\right)\right\|, \\ \left\|x_n(t) - T_j\left(t, u(t)\right)\right\|, \left\|u(t) - S_i\left(t, x_n(t)\right)\right\| \end{array}\right\} \tag{2.7}$$

Further, if T_i, S_j satisfy (iv), then clearly they will satisfy (ii).
On applying the limit $n \to \infty$, in (2.5), (2.6), (2.7), we get

$$\left\|u(t) - T_j\left(t, u(t)\right)\right\| \le \lambda \left\|u(t) - T_j\left(t, u(t)\right)\right\|, \tag{2.8}$$

where $\lambda = \max\{b + c, q, \alpha\} < 1$. Then $T_j\left(t, u(t)\right) = u(t)$. Similarly, we can show that $S_i\left(t, u(t)\right) = u(t)$.

To prove the uniqueness of $u(t)$ in the case (iv), let us assume that $v(t)$ be common fixed point of *S* and *T* other than $u(t)$, then using (iv) we have

$$\begin{aligned} \left\|u(t) - v(t)\right\| &= \left\|S_i\left(t, u(t)\right) - T_j\left(t, v(t)\right)\right\| \\ &\le q \max\left\{\begin{array}{l} \left\|u(t) - v(t)\right\|, \left\|u(t) - S_i\left(t, u(t)\right)\right\|, \\ \left\|v(t) - T_j\left(t, v(t)\right)\right\|, \left\|u(t) - T_j\left(t, v(t)\right)\right\|, \left\|v(t) - S_i\left(t, u(t)\right)\right\| \end{array}\right\} \\ &\le q \left\|u(t) - v(t)\right\|, \end{aligned}$$

which further yields $(1 - q)\left\|u(t) - v(t)\right\| \le 0$.

But $0 < q < 1$, hence $\left\|u(t) - v(t)\right\| < 0$, which is a contradiction as norm is always nonnegative, therefore $u(t) = v(t)$ always.

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