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Effect of heat source/sink on MHD Mixed convection Boundary layer flow on a vertical surface in a porous medium saturated by a nanofluid with suction or injection

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ABSTRACT - *The effect of heat source/sink on steady two-dimensional laminar MHD mixed convection boundary layer flow past a vertical permeable surface embedded in a porous medium saturated by a nanofluid is performed in this study. Numerical solutions of the similarity equations are obtained using the shooting method. Three types of metallic or nonmetallic nanoparticles, namely Copper (Cu), Alumina (Al_2O_3) and Titania (TiO_2) are considered by using a water-based fluid to investigate the effect of the solid volume fraction or nanoparticle volume fraction parameter φ of the nanofluid. The numerical results of the skin friction coefficient and the velocity profiles are presented and discussed. It is found that the imposition of suction is to increase the velocity profiles and to delay the separation of boundary layer, while the injection parameter decreases the velocity profiles. On the other hand, the range of solutions for the injection case is largest for Al_2O_3 nanoparticles and smallest for Cu nanoparticles.*

Keywords - *Boundary Layer, Nanofluid, Mixed Convection, Permeable Surface, Porous Medium, Magnetic field*

I. INTRODUCTION

The research of magnetohydrodynamic (MHD) incompressible viscous flow has many important engineering applications in devices such as power generator, the cooling of reactors, the design of heat exchangers and MHD accelerators. Yih (1997) considered the effect of uniform blowing/suction on forced convective heat transfer of magnetohydrodynamic Hiemenz flow through porous media invoking the model of the porous medium proposed by Raptis and Takhar (1987).

The problem of convection in a porous medium provides one of the basic scenarios for heat transfer theory and thus is of considerable theoretical and practical interest and has been extensively studied. Excellent reviews of the topic can be found in the books by Nield and Bejan (2006); Pop and Ingham (2001); Ingham and Pop (2005) and Vadasz (2008). The most basic problem for natural or free convection in a porous medium past a vertical flat plat was first studied by Cheng and Minkowycz (1997). There are several numerical studies on the mixed convection in a porous media and we mention here those by Harris et al. (2009); Rosali et al. (2011); Imran et al. (2012) and Mukhopadhyay (2012). On the other hand, nanofluids are engineered by suspending nanoparticles with average size below 100 nm in traditional heat transfer fluids such as water, oil and ethylene glycol. Fluids such as water, oil and ethylene glycol are poor heat transfer fluids, since the thermal conductivity of these fluids play important role on the heat transfer coefficient between the heat transfer medium and the heat transfer surface. Choi and Guarino (1995) showed that the addition of small amount (less than 1% by volume) of nanoparticles to conventional heat transfer liquids increased the thermal conductivity of the fluids up to approximately two times. Therefore, the effective thermal conductivity of nanofluids is expected to enhance heat transfer compared to the conventional heat transfer liquids. Some numerical and experimental studies on the forced and natural convection using nanofluids related with differentially heated enclosures have been considered by Jou and Tzeng (2006); Tiwari and Das (2007); Abu-Nada (2008); Oztop and Abu-Nada (2008); The temperature-dependent heat generation/absorption in energy equation is considered by Vajravelu and Rollins (1992). They adopted the model of Foraboschi and Federico (1964) to study the heat transfer in an electrically conducting fluid over a stretching surface.

In many problems, there may be an appreciable temperature difference between the surface and the ambient fluid. This necessitates the consideration of temperature-dependent heat sources or sinks which may exert strong influence on the heat transfer characteristics. The study of heat generation or absorption in moving fluids is important in view of several physical problems such as fluids undergoing exothermic or endothermic chemical reactions. Exact modelling of the internal heat generation or absorption is quite difficult, some simple mathematical models can express its average behaviour for most physical situations. Heat generation or absorption has been assumed to be constant, space-dependent or temperature dependent. Sparrow and Cess (1961) considered temperature dependent heat absorption in their work on steady stagnation point flow and heat transfer. Watanabe(1991) investigated the forced and free mixed convection boundary layer flow with uniform suction or injection on a vertical flat plate. Vajravelu and Nayfeh(1998) reported on the hydromagnetic convection at a cone and a wedge in the presence of temperature-dependent heat generation or absorption effects. Chamkha(1992) reported the non-darcy fully developed mixed convection in a porous medium channel with heat generation/absorption. Chamkha et al. (2003) discussed effects of thermal radiation on MHD forced convection flow adjacent to a non-isothermal wedge in the presence of a heat source or sink. Azim et al. (2010) reported viscous Joule heating MHD–conjugate heat transfer for a vertical flat plate in the presence of heat generation. Alam et al. (2007) investigated the similarity Solutions for hydromagnetic free convective heat and mass transfer flow along a semi-infinite permeable inclined flat plate with heat generation and thermophoresis. Chamkha et al. (2004) studied the mixed convection flow over a vertical plate with localized heating and cooling along with magnetic effects and suction/injection effects. Kandasamy et al. (2010) discussed the effect of temperature-dependent fluid viscosity with thermophoresis and chemical reaction on MHD free convective heat and mass transfer over a porous stretching surface in the presence of heat source/sink. Recently, Sharma et al. (2010) investigated the combined effect of magnetic field and heat absorption on unsteady free convection and heat transfer flow in a micropolar fluid past a semi-infinite moving plate with viscous dissipation using element free Galerkin method.

In the present paper, we have considered the heat source/sink effect on MHD mixed convection boundary layer flow saturated by electrically conducting nanofluids over a vertical permeable flat plate with linear wall temperature proportional to the distance in a porous medium.

II. PROBLEM FORMULATION

In this paper the effect of heat source/sink on the steady two-dimensional laminar MHD mixed convection boundary layer flow past a vertical semi-infinite plate embedded in a porous medium filled with a nanofluid is studied. It is assumed that the free stream velocity and the ambient temperature (far flow from the plate) are U_∞ and T_∞, respectively. It is also assumed that the temperature of the plate is T_w, where $T_w > T_\infty$ corresponds to a heated plate (assisting flow) and $T_w < T_\infty$ corresponds to a cooled plate (opposing flow). It is also assumed that the convecting fluid and the porous medium are in local thermodynamic equilibrium, the viscous dissipation is neglected, the physical properties of the fluid except the density are constant and that the Boussinesq approximation holds. Following the nanofluid equation model proposed by Tiwari and Das (2007) along with the Boussinesq and boundary layer approximations, it is easy to show that the steady boundary layer equations of the present problem are (Ahmad and Pop, 2010):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\mu_{nf}}{\mu_f}\frac{\partial u}{\partial y} = \frac{gK[\varphi\rho_s\beta_s + (1-\varphi)\rho_f\beta_f]}{\mu_f}\frac{\partial T}{\partial y} - \frac{\sigma B_0^2}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho C_p}(T - T_\infty) \tag{3}$$

Subject to the boundary conditions:

$$u = 0\,,\ \ v = v_w\ \ , T = T_w(x)\ \ at\ y = 0$$

$$u \to \infty\ , T \to T_\infty\ \ as\ \ y \to \infty \tag{4}$$

Integrating Equation 2 with the boundary conditions Equation 4, it becomes:

$$\frac{\mu_{nf}}{\mu_f}u = \frac{\mu_{nf}}{\mu_f}U_\infty + \frac{gK[\varphi\rho_s\beta_s + (1-\varphi)\rho_f\beta_f]}{\mu_f}(T - T_\infty) - \frac{\sigma B_0^2}{\rho}(u - U_\infty) \tag{5}$$

Here, x and y are the Cartesian coordinates measured along the plate and normal to it, respectively, u and v are the velocity components along x and y axes, respectively, T is the temperature of the nanofluids, g is the acceleration due to gravity, $v_w(x)$ is the mass transfer velocity with $v_w(x) < 0$ for suction and $v_w(x) > 0$ for injection, ϕ is the nanoparticle volume fraction, μ_f is the dynamic viscosity of the base fluid, β_f and β_s are the coefficients of thermal expansion of the fluid and of the solid, respectively, ρ_f and ρ_s are the densities of the fluid and of the solid fractions, respectively, μ_{nf} is the viscosity of the nanofluid and α_{nf} is the thermal diffusivity of the nanofluid; k is the permeability of the porous medium; σ is the electrical conductivity; B_0 is the externally imposed magnetic field in the y-direction. The induced magnetic field effect is neglected for the small magnetic Reynolds number flow, by Shercliff (1965). It is also assumed that the external electric field is zero, the electric field owing to polarization of charges and the Hall effect are neglected; Q is the volumetric rate of heat generation/absorption; C_p is the heat capacity at constant pressure, and the Thermophysical properties of fluid and nanoparticles are given by Oztop and Abu-Nada (2008):

$$\mu_{nf} = \frac{\mu_f}{(1-\varphi)^{2.5}}, \quad \alpha_{nf} = \frac{K_{nf}}{(\rho C_p)_{nf}}$$

$$(\rho C_p)_{nf} = (1-\varphi)(\rho C_p)_f + \varphi(\rho C_p)_s, \tag{6}$$

$$\frac{K_{nf}}{K_f} = \frac{(K_s + 2K_f) - 2\varphi(K_f - K_s)}{(K_s + 2K_f) + \varphi(K_f - K_s)}$$

Where, $(\rho C_p)_{nf}$ is the heat capacity of the nanofluid as expressed by Khanafer et al. (2003) and Abu-Nada (2008). The viscosity of the nanofluid μ_{nf} can be approximated as the viscosity of a base fluid μ_f containing dilute suspension of fine spherical particles. The effective thermal conductivity of the nanofluid k_{nf} is approximated by the Maxwell-Garnett's model, which is found to be appropriate for studying the heat transfer enhancement using nanofluids (Abu-Nada, 2008). Similarity solutions of Equation 3 and 5 subject to the boundary conditions (4) of the following form:

$$\psi = \alpha_f(2Pe_x)^{1/2}f(\eta), \quad \theta = \frac{(T-T_\infty)}{(T_w-T_\infty)}, \quad \eta = \left(\frac{Pe_x}{2}\right)^{1/2}\left(\frac{y}{x}\right) \tag{7}$$

Where, $Pe_x = U_\infty x / \alpha_f$ is the local Peclet number for the porous medium and ψ is the stream function, which is defined in the usual way as $u = \partial\psi/\partial y$ and $v = \partial\psi/\partial x$. Thus, we have Equation (8):

$$u = U_\infty f'(\eta)$$

$$v = -\frac{1}{2}\frac{\alpha_f(2Pe_x)^{1/2}}{x}[f(\eta) - \eta f'(\eta)] \tag{8}$$

Where, primes denote differentiation with respect to η . In order that Equation 1 to 3 subject to the boundary conditions (4) admits a similarity solution, we have to consider that $v_w(x)$ has the following expression Equation (9):

$$V_w(x) = -\frac{1}{2}\frac{\alpha_f(2Pe_x)^{1/2}}{x}f_0 \tag{9}$$

where, f_0 is the constant mass transfer parameter with $f_0 > 0$ for suction and $f_0 < 0$ for injection. Substituting Equation 6 and 7 into Equation 3 and 5, we obtain the following system of ordinary differential equations:

$$\frac{1}{(1-\varphi)^{2.5}}f' = \frac{1}{(1-\varphi)^{2.5}} + \left[(1-\varphi) + \varphi\left(\frac{\rho_s}{\rho_f}\right)\left(\frac{\beta_s}{\beta_f}\right)\right]\lambda\theta - M^2(f'-1) \tag{10}$$

$$\frac{K_{nf}/K_f}{(1-\varphi) + \varphi\frac{(\rho C_p)_s}{(\rho C_p)_f}}\theta'' + f\theta' + Q\theta = 0 \tag{11}$$

Along with the boundary conditions:

$$f(0) = f_0\,,\quad \theta(0) = 1\,,\quad f'(\infty) \to 1\quad ,\theta(\infty) \to 0$$

Where, λ is the constant mixed convection parameter which is defined as Equation (12):

$$\lambda = \frac{Ra_x}{Pe_x} \tag{12}$$

With $Ra_x = \rho_f g k \beta_f (T_w - T_\infty) x / \alpha_f \mu_f$ being the local Rayleigh number for a porous medium. It is worth mentioning that $\lambda > 0$ corresponds to an assisting flow (heated plate), $\lambda < 0$ corresponds to an opposing flow (cooled plate) and $\lambda = 0$ corresponds to the forced convection flow. $M = B_0\sqrt{\sigma / C\rho}$ is the Hartmann number, $Q = Q_0 / (\rho C_p)_f C$,is the heat source/sink parameter . Further, Equation 10 and 11 can be combined to give single equation:

$$\frac{K_{nf}/K_f}{(1-\varphi) + \varphi\frac{(\rho C_p)_s}{(\rho C_p)_f}}f''' + ff'' + Qf' = 0 \tag{13}$$

Subject to the boundary condition Equation (14):

$$f(0) = f_0 \quad , \quad f'(\infty) \to 1$$

$$\frac{1}{(1-\varphi)^{2.5}} f'(0) = \frac{1}{(1-\varphi)^{2.5}} + \left[(1-\varphi) + \varphi\left(\frac{\rho_s}{\rho_f}\right)\left(\frac{\beta_s}{\beta_f}\right)\right]\lambda - M^2\left(f'(0) - 1\right) \tag{14}$$

The physical quantity of interest is the skin friction coefficient C_f, which is defined as:

$$C_f = \frac{\tau_w}{\rho_f U_\infty^2} \tag{15}$$

where τ_w is the skin friction or the shear stress at the surface of the plate, which is given by:

$$\tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{16}$$

Substituting Equation 7 into Equation 15 and 16, we obtain Equation 17:

$$(2Pe_x)^{1/2} C_f = \frac{1}{(1-\varphi)^{2.5}} f''(\theta) \tag{17}$$

III. RESULTS AND DISCUSSION

The nonlinear ordinary differential Equation 13 subject to the boundary conditions 14 was solved numerically using the shooting method. This wellknown technique is an iterative algorithm which attempts to identify appropriate initial conditions for a related Initial Value Problem (IVP) that provides the solution to the original Boundary Value Problem (BVP). The shooting method is based on MAPLE “dsolve” command and MAPLE implementation, “shoot” (Meade *et al.*, 1996). The results are given to carry out a parametric study showing the influences of the non-dimensional parameters, namely the mixed convection parameter λ and the constant suction/injection parameter f_0, the Hartmann number M, the heat source/sink parameter γ. Following Oztop and Abu-Nada (2008) we have considered the range of nanoparticles volume fraction ϕ as $0 < \phi < 0.2$. The thermophysical properties of fluid and nanoparticles (Cu, Al_2O_3, TiO_2) used in this study are given in Table 1. In order to validate the accuracy of the numerical method used, the present results for the skin friction coefficient $f''(0)$ when $\phi = 0.1$ and $\phi = 0.2$,M=0.5,Q=0.5 for Cu nanoparticles and various values of λ are compared with those of yasin as shown in Table 2 and 3, respectively. It is clearly seen that the comparison shows very good agreement. Table 2 and 3 also illustrate the influence of the suction and injection parameter $f_0 = 0.5$ (suction), 0 (impermeable) and -0.5 (injection) for Cu nanoparticles and various values of λ. The results indicate that the imposition of suction $(f_0 > 0)$ at the surface has the tendency to increase the skin friction coefficient $f''(0)$ but for the case of surface injection $(f_0 < 0)$, the skin friction coefficient $f''(0)$ decreases, which show a favorable agreement with the previous investigations for the case of impermeable surface $(f''(0))$. Figure 1 shows the variation of the skin friction coefficient $(2Pe_x)^{1/2} C_f$ with λ for different types of nanoparticles (Cu, Al_2O_3, TiO_2) when $\phi = 0.1$. This figure shows that it is possible to get dual solutions of the similarity Equation 13 subjected to boundary conditions 14 for the opposing flow case $(\lambda < 0)$ with upper and lower branch solutions. Dual solutions exist for $\lambda_c < \lambda > 0$, a unique solution exists for $\lambda_c = \lambda < 0$ and no solutions exist for $\lambda_c = \lambda > 0$, where λ_c is the critical value of λ for which the solution exists. As in similar physical situations, we postulate that the upper branch solutions are physically stable and occur in practice, whilst the lower branch solutions are not physically realizable. This postulate can be verified by performing a stability analysis but this is beyond the scope of the present paper. On the other hand, it is also shown in Fig. 1 that suction $(f_0 > 0)$ delays separation compared to the impermeable surface or injection $(f_0 < 0)$ cases. This is true for all the three nanoparticles (Cu, Al2O3, TiO2) considered. The variation of the skin friction coefficient $(2\,Pe_x)^{1/2} C_f$ with suction/injection parameter f_0 when $\phi = 0.1$ and $\lambda = -1.6$ are presented in Fig. 2. This figure supports the dual nature of the solutions to the boundary value problem (14) and (15). For this value of λ, there is a critical value f_{0c} of f_0, at which there is a saddle-node bifurcation, with two solutions for $(f_0 > f_{0c})$ and no solutions for $f_0 < f_{0c} < 0$. This indicates that injection (having $(f_0 < 0)$) limits the existence of solutions, whereas no such limit appears for suction $(f_0 > 0)$, with both branches of solutions continuing to large values of $f_0 > 0$ (suction). Based on our computations, the values of f_{0c} are f_{0c} = -0.51515l for Al2O3, f_{0c} = -0.49825 for TiO2 and f_{0c} = -0.33955 for Cu. This shows that the range of solutions for the injection $(f_0 < 0)$ case is largest for Al_2O_3 nanoparticles and smallest for Cu nanoparticles.

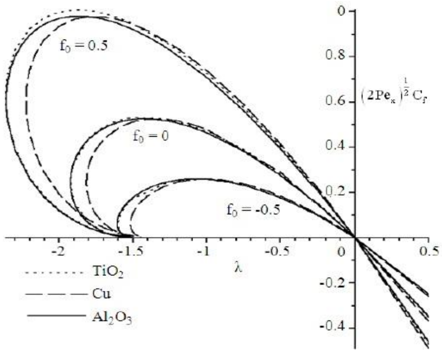

Fig. 1. Variation of the skin friction coefficient $(2\,Pe_x)^{1/2} C_f$ with □ □ for different types of nanoparticles when □ □ = 0.1 and various values of f_0

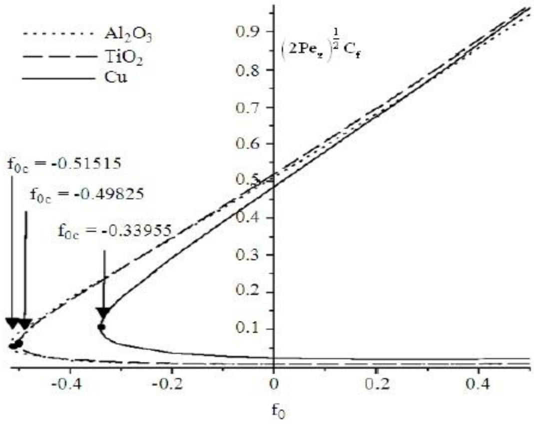

Fig. 2. Variation of the skin friction coefficient $(2\,Pe_x)^{1/2} C_f$ with f_0 for different types of nanoparticles when □ □ = 0.1 and □ □ =-1.6

Figure 3-5(a-b-c-d) show the Comparison of the velocity profiles whit the effect of heat source/sink and magnetic field when 0≤M≤1, 0≤Q≤0.5 and $\phi = 0.1$ for different types of nanoparticles, namely Cu, Al_2O_3, TiO_2 respectively. The dashed line refers to the Compare article (Yasin 2013). As you see in Figs 3-(a,b,c,d) which refer to CU nanoparticle, the maximum point for both articles, the present work and yasin (2013) is less than 5 in x direction. Fig (3-c) shows that whit Q=0, the velocity profiles are in the same maximum point, it means that, the present results are in good agreement with those reported by yasin and we also seen that ,the velocity profiles decrease whit lower number of M, and the maximum point increase whit higher number of Q (the heat source/sink parameter).Figures 4-(a,b) which refers to Al_2o_3 nanoparticle have the similar behavior whit cu nanoparticle.

Fig 5(a,b,c,d) also show the results for TiO_2 nanoparticle. As figures show the higher number of M (Hartman number) increase the velocity profiles,and the lower number of Q decrease the velocity profiles, and the suction parameter increases the velocity profiles and the injection parameter decreases the velocity profiles. Finally, it is worth mentioning that all the velocity profiles presented in Fig. 3-5 satisfy the far field boundary conditions (15) asymptotically.

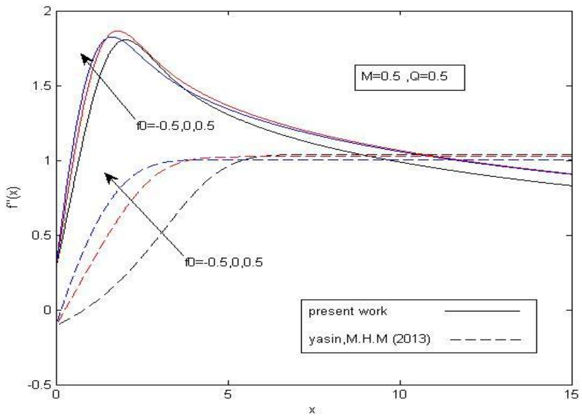

Fig.(3-a) .Cu nanoparticles whit M=0.5 , Q=0.5

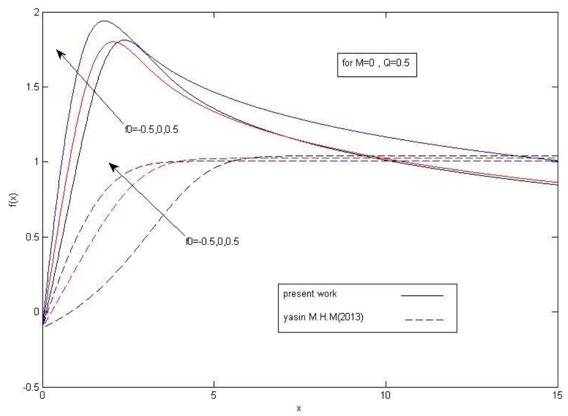

Fig.(3-b). Cu nanoparticles whit M=0, Q=0.

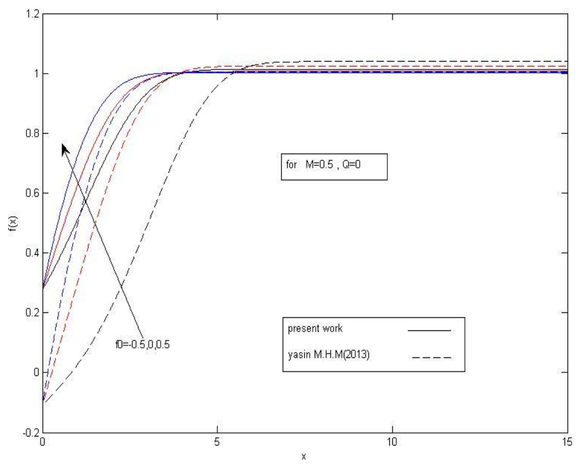

Fig.(3-c) . Cu nanoparticles whit M=0.5,Q=0

Fig. 3(a,b,c). Comparison of the Velocity profiles $f'(\eta)$ for Cu nanoparticles whit yasin (2013) when $\phi = 0.1$, $\lambda = -1.6$ and various values of f_0 ,M,Q

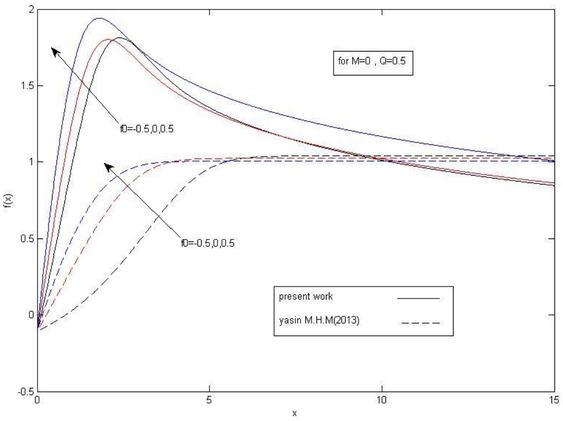

Fig.(4-a) . AL_2O_3 nanoparticles whit M=0 , Q=0.5

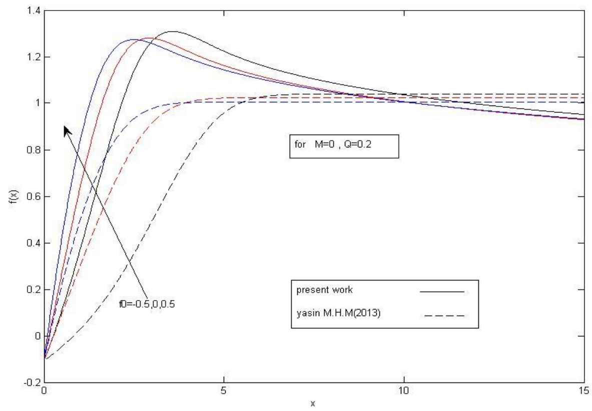

Fig.(4-b) . AL_2O_3 nanoparticles whit M=0 , Q=0.2

Fig. 4(a,b). Comparison of the Velocity profiles $f'(\eta)$ for AL_2O_3 nanoparticles whit yasin (2013) when $\phi = 0.1$, $\lambda = -1.6$ and various values of f_0 ,M,Q

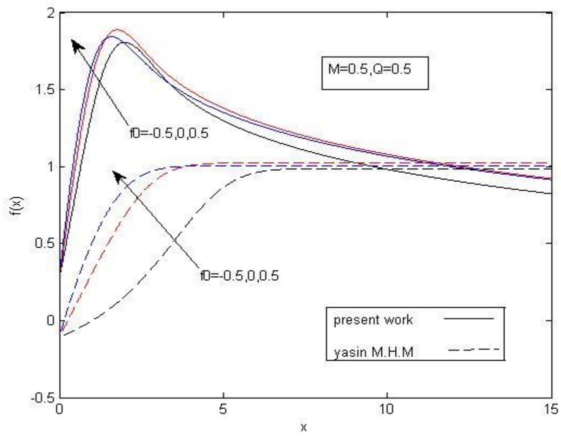

Fig.(5-a) . TiO_2 nanoparticles whit M=0.5 , Q=0.5

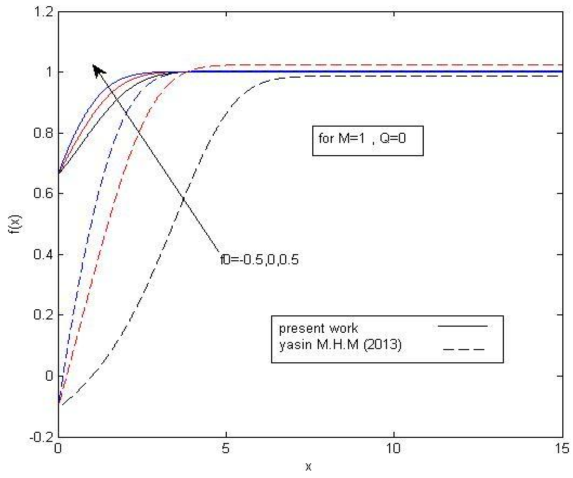

Fig.(5-b) . TiO_2 nanoparticles whit M=1 , Q=0

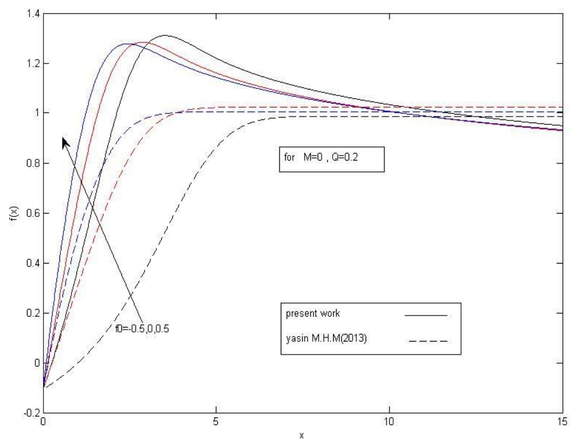

Fig.(5-c) . TiO_2 nanoparticles whit M=0 , Q=0.2

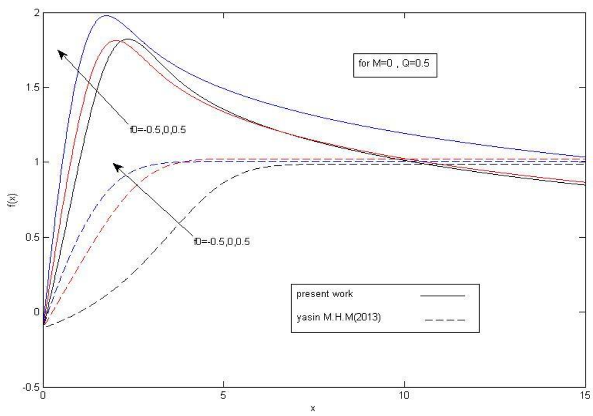

Fig.(5-d) . TiO_2 nanoparticles whit M=0 , Q=0.5

Fig. 5(a,b,c,d). Comparison of the Velocity profiles $f'(\eta)$ for TiO_2 nanoparticles whit yasin (2013) when $\phi = 0.1$, $\lambda = -1.6$ and various values of f_0 ,M,Q

Table 1. Thermophysical properties of fluid and nanoparticles (Oztop and Abu-Nada, 2008)

Physical properties	Fluid phase (water)	Cu	Al2O3	TiO2
Cp (J/kg K)	4179.000	385	765	686.2000
ρ **(Kg/m3)**	999.100	8933	3970	4250.000
K (W/mK)	0.613	400	40	8.9538

Table 2. Values of $f''(0)$ for Cu nanoparticles when □ □ = 0.1, M=0.5,Q=0.5

	Yasin M.H.M(2013)	**present**		
λ	F_0**=0**	F_0**=-0.5**	F_0**=0**	F_0**=0.5**
-1.45	0.39852	1.763	1.835	1.796
-1.50	0.39263	1.769	1.843	1.806
-1.516				
-1.55	0.38391	1.799	1.850	1.816
-1.60	0.37176	1.789	1.823	1.822
-1.65	0.35523			
-1.70	0.33259	1.792	1.843	1.884
-1.85	0.30004	1.689		
-1.90	0.24202	1.858	1.856	1.917
-1.95	0.18714			
-2.00				
-2.225				

Table 3. Values of $f''(0)$ for Cu nanoparticles when □ □ = 0.2,M=0.5 , Q=0.5

	Yasin M.H.M(2013)	**present**		
λ	F_0**=0**	F_0**=-0.5**	F_0**=0**	F_0**=0.5**
-1.75	0.34746	1.787	1.741	1.715
-2.00	0.34528	2.334	1.738	1.714
-2.14460				
-2.20	0.32409	0.8833	1.736	1.711
-2.25	0.31476	0.8805	1.736	1.717
-2.30	0.30311			
-2.35	0.28845	0.3941	1.711	1.719
-2.40	0.26949			
-2.45	0.24321	2.166	1.711	1.724
-2.50	0.19559			
-2.50987	0.16214			
-2.60				

IV. CONCLUSION

The aim of this present paper is to investigate numerically heat source/sink effect on the steady two-dimensional laminar MHD mixed convection Hiemenz flow over a vertical permeable flat plate placed in a porous medium and saturated by a nanofluid as considered by Ahmad and Pop (2010) and yasin (2013). We have extended the previous work by taking into consideration the effects of suction or injection and magnetic field and the effects of heat source /sink,with permeable surface. Further, the governing equations are transformed into ordinary differential equations and are then solved numerically using the shooting method. The effects of the suction or injection and the heat source/sink parameter, the mixed convection and magnetic field parameter and the nanoparticle volume fraction parameter on the flow and heat transfer characteristics are studied. In general, imposition of suction is to increase the velocity profiles and to delay the separation of boundary layer, while the injection parameter decreases the velocity profiles. On the other hand, the higher number of M (Hartman number) increase the velocity profiles,and the lower number of Q (heat source/sink parameter) decrease the velocity profiles and when the Q=M=0 the diagrams are the same.the range of solutions for the injection case is largest for Al_2O_3 nanoparticles and smallest for Cu nanoparticles.

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