Research Inventy: International Journal Of Engineering And Science
Issn: 2278-4721, Vol. 2, Issue 4 (February 2013), Pp 025-30
Www.Researchinventy.Com

Fuzzy W- Super Continuous Mappings

[1]M. K. Mishra , [2]M. SHUKLA[3] S.Kamali
1,Professor, EGS PEC Nagapattinam
[2,]Asst. Prof. AGCW KARAIKAL
[3,]Asst.Prof Anna Univ.Thirukali Campus

Abstract: *The purpose of this paper to introduce and study the concepts of fuzzy w-super closed sets and fuzzy w- super continuous mappings in fuzzy topological spaces.*
Keywords: *fuzzy super closure fuzzy super interior, fuzzy super open set , fuzzy super closed set , fuzzy w-super closed set , fuzzy g-super closed set, fuzzy g- super open set, fuzzy sg- super closed set, fuzzy sg- super open set, fuzzy gs- super closed set.*

I. Preliminaries

Let X be a non empty set and I= [0,1]. A fuzzy set on X is a mapping from X in to I. The null fuzzy set 0 is the mapping from X in to I which assumes only the value is 0 and whole fuzzy sets 1 is a mapping from X on to I which takes the values 1 only. The union (resp. intersection) of a family $\{A_\alpha: \alpha\in\Lambda\}$of fuzzy sets of X is defined by to be the mapping sup A_α (resp. inf A_α) . A fuzzy set A of X is contained in a fuzzy set B of X if A(x) $\leq$ B(x) for each $x\in X$. A fuzzy point x_β in X is a fuzzy set defined by x_β (y)=β for y=x and x(y) =0 for y $\neq$ x, $\beta\in[0,1]$ and y $\in$ X .A fuzzy point x_β is said to be quasi-coincident with the fuzzy set A denoted by $x_{\beta q}A$ if and only if β + A(x) > 1. A fuzzy set A is quasi –coincident with a fuzzy set B denoted by A_qB if and only if there exists a point $x\in X$ such that A(x) + B(x) > 1 .A $\leq$ B if and only if $\rceil(A_qB^c)$.A family τ of fuzzy sets of X is called a fuzzy topology [2] on X if 0,1 belongs to τ and τ is super closed with respect to arbitrary union and finite intersection .The members of τ are called fuzzy super open sets and their complement are fuzzy super closed sets. For any fuzzy set A of X the closure of A (denoted by cl(A)) is the intersection of all the fuzzy super closed super sets of A and the interior of A (denoted by int(A))is the union of all fuzzy super open subsets of A.

Defination1.1 [5]: Let (X,τ) fuzzy topological space and $A\subseteq X$ then

1. Fuzzy Super closure $scl(A)=\{x\in X:cl(U)\cap A\neq\phi\}$
2. Fuzzy Super interior $sint(A) =\{x\in X:cl(U)\leq A\neq\phi\}$

Definition 1.2[5]: A fuzzy set A of a fuzzy topological space (X,τ) is called:
(a) Fuzzy super closed if scl(A) $\leq$ A.
(b) Fuzzy super open if 1-A is fuzzy super closed sint(A)=A

Remark 1.1[5]: Every fuzzy closed set is fuzzy super closed but the converses may not be true.

Remark 1.2[5]: Let A and B are two fuzzy super closed sets in a fuzzy topological space $(X,\Im)$, then A $\cup$ B is fuzzy super closed.

Remark 1.3[5]: The intersection of two fuzzy super closed sets in a fuzzy topological space $(X,\Im)$ may not be fuzzy super closed.

Definition 1.3[1,5,6,7]: A fuzzy set A of a fuzzy topological space (X,τ) is called:

(a) fuzzy semi super open if there exists a super open set O such that O$\leq$ A$\leq$ cl(O).
(b) fuzzy semi super closed if its complement 1-A is fuzzy semi super open.

Remark 1.4[1,5,7]: Every fuzzy super open (resp. fuzzy super closed) set is fuzzy semi super open (resp. fuzzy semi super closed) but the converse may not be true .

Definition 1.4[5]: The intersection of all fuzzy super closed sets which contains A is called the semi super closure of a fuzzy set A of a fuzzy topological space (X,τ) . It is denoted by scl(A).

Definition 1.5[3,11,8,9,10]: A fuzzy set A of a fuzzy topological space (X,τ) is called:

1. fuzzy g- super closed if $cl(A) \leq G$ whenever $A \leq G$ and G is super open.
2. fuzzy g- super open if its complement 1-A is fuzzy g- super closed.
3. fuzzy sg- super closed if $scl(A) \leq O$ whenever $A \leq O$ and O is fuzzy semi super open.
4. fuzzy sg- super open if if its complement 1-A is sg- super closed.
5. fuzzy gs- super closed if $scl(A) \leq O$ whenever $A \leq O$ and O is fuzzy super open.
6. fuzzy gs- super open if if its complement 1-A is gs- super closed.

Remark 1.5[10,11]: Every fuzzy super closed (resp. fuzzy super open) set is fuzzy g- super closed (resp. fuzzy g- super open) and every fuzzy g-super closed (resp. fuzzy g-super open) set is fuzzy gs-super closed (resp. gs – super open) but the converses may not be true.

Remark 1.6[10,11]: Every fuzzy semi super closed (resp. fuzzy semi super open) set is fuzzy sg-super closed (resp. fuzzy sg-super open) and every fuzzy sg-super closed (resp. fuzzy sg-super open) set is fuzzy gs-super closed (resp. gs – super open) but the converses may not be true .

Definition 1.5[1,3,9,10,11] :A mapping from a fuzzy topological space (X,τ) to a fuzzy topological space (Y,Γ) is said to be :

1. fuzzy super continuous if $f^{-1}(G)$ is fuzzy super open set in X for every fuzzy super open set G in Y.
2. Fuzzy super closed if the inverse image of every fuzzy super closed set of Y is fuzzy super closed in X.
3. fuzzy semi super continuous if $f^{-1}(G)$ is fuzzy semi super open set in X for every fuzzy super open set G in Y.
4. fuzzy irresolute if the inverse image of every fuzzy semi super open set of Y is fuzzy semi super open in X.
5. fuzzy g-super continuous if $f^{-1}(G)$ is fuzzy g-super closed set in X for every fuzzy super closed set G in Y.
6. fuzzy sg-super continuous if $f^{-1}(G)$ is fuzzy sg-super closed set in X for every fuzzy super closed set G in Y.
7. fuzzy gs-super continuous if $f^{-1}(G)$ is fuzzy gs-super closed set in X for every fuzzy super closed set G in Y.

Remark 1.7[1, 11]: Every fuzzy super continuous mapping is fuzzy g-super continuous (resp. fuzzy semi super continuous) but the converse may not be true.

Remark 1.8[10, 11]: Every fuzzy g-super continuous mapping is fuzzy gs-super continuous but the converses may not be true.

Remark 1.9[10, 11]: Every fuzzy semi super continuous mapping is fuzzy sg-super continuous and every sg-super continuous mapping is gs-super continuous but the converses may not be true.

Definition 1.6[10, 11]: A fuzzy topological space (X,τ) is said to be fuzzy $T_{1/2}$ (resp. fuzzy semi $T_{1/2}$) if every fuzzy g-super closed (resp. fuzzy sg-super closed) set in X is fuzzy super closed (resp. fuzzy semi super closed).

Definition 1.7: A fuzzy set A of a fuzzy topological space (X,τ) is called fuzzy w-super closed if $cl(A) \leq U$ whenever $A \leq U$ and U is fuzzy semi super open.

Remark 1.9: Every fuzzy super closed set is fuzzy w-super closed but its converse may not be true. For,

Example2.1: Let $X = \{a, b\}$ and the fuzzy sets A and U are defined as follows:

$A(a)= 0.5$, $A(b)=0.5$ $U(a)= 0.5$,$U(b)=0.4$

Let $\Im = \{\mathbf{0}, \mathbf{1}, U\}$ be a fuzzy topology on X. Then the fuzzy set A is fuzzy w-super closed but it is not fuzzy super closed.

Remark1.10: Every fuzzy w-super closed set is fuzzy g-super closed but the converse may not be true. For,

Example2.: Let X = {a, b} and the fuzzy sets A and U are defined as follows

U(a)= 0.7 , U(b)=0.6 ,A(a)= 0.6 ,A(b)=0.7

Let $\Im = \{0, 1, U\}$ be a fuzzy topology on X. Then the fuzzy set A ,is fuzzy g-super closed but it is not fuzzy w-super closed.

Remark1.11: Every fuzzy w-super closed set is fuzzy sg-super closed, but the converse may not be true. For,

Example2.3: Let X = {a, b} and the fuzzy sets A and U be defined as follows:

A (a) = 0.5, A(b)= 0.3 ,U (a) = 0.5, U(b)= 0.4.

Let $\Im = \{0, \mathbf{1}, U\}$ be a fuzzy topology on X. Then the fuzzy set A is fuzzy sg-super closed but it is not fuzzy w-super closed.

Remark1,12: If A and B are fuzzy w-super closed sets in a fuzzy topological space (X,τ) then $A \cup B$ is fuzzy w-super closed.

Remark1.13: The intersection of any two fuzzy w-super closed sets in a fuzzy topological space (X,τ) may not be fuzzy w-super closed for,

Example2.4: Let X = {a, b, c} and the fuzzy sets U, A and B of X are defined as follows:

U (a) =1, U (b) =0, U(c) =0, A (a) =1 , A (b) =1, A(c) =0, B (a) =1, B (b) =0, B(c) =1

Let $\tau = \{0, U, 1\}$ be a fuzzy topology on X. Then A and B are fuzzy w-super closed set in (X,τ) but $A \cap B$ is not fuzzy w-super closed.

Remark1.14 : Let $A \le B \le cl(B)$ and A is fuzzy w-super closed set in a fuzzy topological space (X,τ), then B is fuzzy w-super closed.

Defination1.7: A fuzzy set A of a fuzzy topological space (X,τ) is called fuzzy w-super open if and only if 1-A is fuzzy w-super closed.

Remark1.15 : Every fuzzy super open (resp. fuzzy g-super open) set is fuzzy w-super open .But the converse may not be true. For the fuzzy set B defined by B(a) = 0.5, B(b) = 0.5 in the fuzzy topological space (X,τ) of example (2.1) is fuzzy w-super open but it is not fuzzy super open and the fuzzy set C defined by C(a) = 0.4, C(b)= 0.3 in the fuzzy topological space (X,τ) of example (2.2) is fuzzy w-super open but it is not fuzzy g-super open .

Remark1.16: A fuzzy set A of a fuzzy topological space (X,τ) is fuzzy w-super open if and only if $F \le int(A)$ whenever $F \le A$ and F is fuzzy semi super closed .

Remark1.17: Let A be a fuzzy w-super open subset of a fuzzy topological space (X,τ) and $int(A) \le B \le A$ then B is fuzzy w-super open.

Defination1.8:A fuzzy topological space (X,τ) is said to be fuzzy semi super normal if for every pair of fuzzy semi super closed sets A and B of X such that $A \le (1\text{-}B)$, there exists fuzzy semi super open sets U and V such that $A \le U$, $B \le V$ and $\rceil(U_qF)$.

Remark1.18 : If F is fuzzy regular super closed and A is fuzzy w-super closed sub set of a fuzzy semi super normal space (X,τ) and $\rceil(A_q F)$.Then there exists fuzzy super open set U and V such that $cl(A) \le U$, $F \le V$ and $\rceil(U_qV)$.

Remark1,19: Let A be a fuzzy w-super closed set in a fuzzy topological space (X,τ) and $f: (X,\tau)\to(Y,\sigma)$ is a fuzzy irresolute and fuzzy super closed mapping. Then f (A) is fuzzy w-super closed in Y.

Definition1.9: A collection $\{A_i : i\in\Lambda)$ of fuzzy w-super open sets in a fuzzy topological space (X,τ) is called a fuzzy w-super open cover of a fuzzy set B of X if $B \leq \cup\{A_i : i\in\Lambda)$.

Definition1.10: A fuzzy topological space (X,τ) is called w-compact if every fuzzy w-super open cover of X has a finite sub cover.

Definition1.11: A fuzzy set B of B of a fuzzy topological space (X,τ) is said to be fuzzy w- super compact relative to X if for every collection $\{A_i: i \in \Lambda\}$ of fuzzy w-super open sub set of X such that $B \leq \cup\{A_i :I \in \Lambda)$ there exists a finite subset Λ_0 of Λ such that $B\leq \cup\{A_i: i \in \Lambda_0)$.

Definition1.12: A crisp subset B of a fuzzy topological space (X,τ) is said to be fuzzy w- super compact if B is fuzzy w- super compact as a fuzzy subspace of X.

Remark1.20: A fuzzy w-super closed crisp subset of a fuzzy w- super compact space is fuzzy w- super compact relative to X.

II. Fuzzy W-Super Continuous Mappings

The present section investigates and a new class of fuzzy mappings which contains the class of all fuzzy super continuous mappings and contained in the class of all fuzzy g-super continuous mappings.

Definition 2.1: A mapping $f: (X,\tau)\to(Y,\sigma)$ is said to be fuzzy w-super continuous if the inverse image of every fuzzy super closed set of Y is fuzzy w-super closed in X.

Theorem 2.1: A mapping $f: (X,\tau)\to(Y,\sigma)$ is fuzzy w-super continuous if and only if the inverse image of every fuzzy super open set of Y is fuzzy w-super open in X.

Proof: It is obvious because $f^{-1}(1-U)=1-f^{-1}(U)$ for every fuzzy set U of Y.

Remark 2.1: Every fuzzy super continuous mapping is fuzzy w-super continuous, but the converse may not be true for,

Example 2.1: Let X = {a, b} and Y = {x, y} and the fuzzy sets U and V are defined as follows

U (a) = 0.5, U (b) = 0.4,V(x) = 0.5, V(y) = 0.5,

Let $\tau = \{0,U,1\}$ and $\sigma = \{0,V,1\}$, be fuzzy topologies on X and Y respectively .Then the mapping $f: (X,\tau)\to(Y,\sigma)$ defined by f(a) = x and f(b) = y is fuzzy w-super continuous but not fuzzy super continuous.

Remark 2.2: Every fuzzy w-super continuous mapping is fuzzy g-super continuous, but the converse may not be true for,

Example 2.2: Let X = {a, b} and Y = {x, y} and the fuzzy sets U and V are defined as follows

U (a) = 0.7, U (b) = 0.6,V(x) = 0.6, V(y) = 0.7,

Let $\tau= \{0,U,1\}$ and $\sigma = \{0,V,1\}$, be fuzzy topologies on X and Y respectively. Then the mapping $f: (X,\tau)\to(Y,\sigma)$ defined by f(a) = x and f(b) = y is fuzzy g-super continuous but not fuzzy w- super continuous.

Remark 2.3: Every fuzzy w-super continuous mapping is fuzzy sg-super continuous, but the converse may not be true for,

Example 2.3: Let X = {a, b} and Y={x, y} and the fuzzy sets U and V are defined as follows

U (a) =0.5, U (b) = 0.4 ,V(x) = 0.5, V(y) = 0.3,

Let $\tau=\{0,U,1\}$ and $\sigma=\{0,V,1\}$, be fuzzy topologies on X and Y respectively. Then the mapping $f: (X,\tau)\to(Y,\sigma)$ defined by f(a) = x and f(b) = y is fuzzy sg-super continuous but not fuzzy w-super continuous.

Remark 2.4: Remarks 1.4, 1.5, 1.6, 3.1, 3.2 and 3.3 reveals the following diagram of implications:

fuzzy super continuous $\Rightarrow$ fuzzy w-super continuous $\Rightarrow$ fuzzy g-super continuous

$\Downarrow$ $\Downarrow$ $\Downarrow$

fuzzy semi super continuous $\Rightarrow$ fuzzy sg-super continuous $\Rightarrow$ fuzzy gs-super continuous

Theorem 2.2: If f: $(X,\tau)\to(Y,\sigma)$ is fuzzy w-super continuous then for each fuzzy point x_β of X and each fuzzy super open set V of Y such that $f(x_\beta) \in V$ then there exists a fuzzy w-super open set U of X such that $x_\beta \in U$ and $f(U) \leq V$.

Proof: Let x_β be a fuzzy point of X and V is fuzzy super open set of Y such that $f(x_\beta) \in V$, put $U = f^{-1}(V)$.Then by hypothesis U is fuzzy w-super open set of X such that $x_\beta \in U$ and $f(U) = f(f^{-1}(V)) \leq V$.

Theorem 2.3: If f: $(X,\tau)\to(Y,\sigma)$ is fuzzy w-super continuous then for each point $x_\beta \in X$ and each fuzzy super open set V of Y such that $f(x_\beta)_q V$ then there exists a fuzzy w-super open set U of X such that $x_{\beta q}U$ and $f(U) \leq V$.

Proof: Let x_β be a fuzzy point of X and V is fuzzy super open set such that $f(x_\beta)_q V$. Put $U= f^{-1}(V)$.Then by hypothesis U is fuzzy w-super open set of X such that $x_{\beta q}U$ and $f(U) = f(f^{-1}(V)) \leq V$.

Definition 2.5: Let (X,τ) be a fuzzy topological space. The w- super closure of a fuzzy set A of X denoted by wscl(A) is defined as follows:

$wscl(A) = \wedge\{ B: B \geq A, B$ is fuzzy w-super closed set in X $\}$

Remark 2.3: It is clear that, $A \leq gscl(A) \leq wscl(A) \leq scl(A)$ for any fuzzy set A of X.

Theorem2.4: A mapping f: $(X,\tau)\to(Y,\sigma)$ is fuzzy w-super continuous then $f(wscl(A)) \leq scl(f(A))$ for every fuzzy set A of X.

Proof: Let A be a fuzzy set of X. Then cl(f(A)) is a fuzzy super closed set of Y. Since f is fuzzy w-super continuous $f^{-1}(cl(f(A))$ is fuzzy w-super closed in X. Clearly $A \leq f^{-1}(cl(f(A)))$. Therefore $wcl(A) \leq wcl(f^{-1}(cl(f(A))) = f^{-1}(cl(f(A)))$. Hence $f(wcl(A)) \leq cl(f(A))$.

Definition 2.6: A fuzzy topological space (X,τ) is said to be fuzzy w-$T_{1/2}$ if every fuzzy w-super closed set in X is fuzzy semi super closed.

Theorem 2.5: A mapping f from a fuzzy w-$T_{1/2}$ space (X,τ) to a fuzzy topological space (Y,σ) is fuzzy super continuous if and only if it is fuzzy w-super continuous.

Proof: Obvious.

Remark 2.4: The composition of two fuzzy w-super continuous mappings may not be fuzzy w-super continuous. For

Example 2.2: Let X = {a, b} , Y = {x, y} , Z= {p, q}and the fuzzy sets U ,V and W defined as follows: U (a) =0.5 , U (b) = 0.4,V(x) = 0.5 , V(y) = 0.3 ,W (p) = 0.6 , W (q) = 0.4 Let $\tau = \{0,U,1\}$ and $\sigma = \{0,V,1\}$ and $\mu = \{0,W,1\}$be the fuzzy topologies on X ,Y and Z respectively. Then the mapping f: $(X,\tau)\to(Y,\sigma)$ defined by f(a) = x and f(b) = y and the mapping g:$(Y,\sigma)\to(Z,\mu)$ defined by g(x) = p, and g(y) = q. Then f and g are w-super continuous but gof is not fuzzy w-super continuous. However,

Theorem 2.6: If f: $(X,\tau)\to(Y,\sigma)$ is fuzzy w-super continuous and g:$(Y,\sigma)\to(Z,\mu)$ is fuzzy super continuous . Then gof : $(X,\tau)\to (Z,\mu)$ is fuzzy w-super continuous.

Proof: Let A be a fuzzy super closed in Z then g^{-1} (A) is fuzzy super closed in Y, because g is super continuous .Therefore $(gof)^{-1}(A) = f^{-1}(g^{-1}(A))$ is fuzzy w-super closed in X. Hence gof is fuzzy w-super continuous**.**

Theorem 2.7: If f: $(X,\tau)\rightarrow(Y,\sigma)$ is fuzzy w-super continuous and g:$(Y,\sigma)\rightarrow(Z,\mu)$ is fuzzy super continuous is fuzzy g-super continuous and (Y,σ) is fuzzy $T_{1/2,}$ then gof: $(X,\tau)\rightarrow(Z,\mu)$ is fuzzy w-super continuous.

Proof: Let A be a fuzzy super closed set in Z then $g^{-1}(A)$ is fuzzy g-super closed set in Y because g is g-super continuous. Since Y is $T_{1/2}$, $g^{-1}(A)$ is fuzzy super closed in Y. And so, $(gof)^{-1}(A) = f^{-1}(g^{-1}(A))$ is fuzzy w-super closed in X. Hence gof: $(X,\tau)\rightarrow(Z,\mu)$ is fuzzy w-super continuous

Theorem 2.8: A fuzzy w-super continuous image of a fuzzy w-compact space is fuzzy compact.

Proof: Let f: $(X,\tau)\rightarrow(Y,\sigma)$ is a fuzzy w-super continuous mapping from a fuzzy w-compact space (X,τ) on to a fuzzy topological space (Y,σ). Let $\{A_i:i\in\Lambda\}$ be a fuzzy super open cover of Y. Then $\{f^{-1}(A_i : i\in\Lambda)\}$ is a fuzzy w-super open cover of X. Since X is fuzzy w- super compact it has a finite fuzzy sub cover say $f^{-1}(A_1)$, $f^{-1}(A_2)$,…………,$f^{-1}(A_n)\}$. Since f is on to $\{A_1,A_2,\ldots..A_n\}$ is an super open cover of Y. Hence (Y,σ) is fuzzy compact.

Definition 2.7: A fuzzy topological space X is said to be fuzzy w- super connected if there is no proper fuzzy set of X which is both fuzzy w-super open and fuzzy w-super closed.

Remark 2.5: Every fuzzy w- super connected space is fuzzy super connected, but the converse may not be true for the fuzzy topological space (X,τ) in example 2.1 is fuzzy super connected but not fuzzy w- super connected.

Theorem 2.9: If f: $(X,\tau)\rightarrow(Y,\sigma)$ is a fuzzy w-super continuous surjection and X is fuzzy w- super connected then Y is fuzzy super connected.

Proof: Suppose Y is not fuzzy super connected .Then there exists a proper fuzzy set G of Y which is both fuzzy super open and fuzzy super closed .Therefore $f^{-1}(G)$ is a proper fuzzy set of X, which is both fuzzy w-super open and fuzzy w-super closed in X, because f is fuzzy w-super continuous surjection. Hence X is not fuzzy w-connected, which is a contradiction.

Reference

[1] Azad K.K on fuzzy semi continuity, fuzzy Almost continuity and fuzzy weakly continuity , J. Math. Anal. Appl. 82(1981),14-32.

[2] Balchandran K, Sundram P and Maki H., on Generalized Super continuous Map in Topological Spaces , Mem. Fac. sci. Kochi Univ.(Math)12(1991),5-13.

[3] Chang C.L. fuzzy Topological Spaces J. Math Anal. Appl.24(1968),182-190.

[4] El-Shafei M. E. and Zakari A. semi-generalized super continuous mappings in fuzzy topological spaces J. Egypt. Math. Soc.15(1)(2007) , 57{67.

[5] Mishra M.K. ,et all on " Fuzzy super closed set" Accepted.

[6] Mishra M..K. ,et all on " Fuzzy super continuity" (Accepted)

[7] Mishra M.K., Shukla M. "Fuzzy Regular Generalized Super Closed Set" (Accepted).

[8] Pu P. M. and Liu Y. M. Fuzzy Topology I: Neighborhood structure of a fuzzy point and More Smith Convergence, J. Math. Anal Appl.76(1980)571-599.

[9] Pushpaltha A. studies on Generalization of Mappings in Topological Spaces Ph.D. Thesis Bharathiyar University Coimbotore (2000).

[10] Sundram P. and M. Shekh John, On w-super closed sets in topology, Acta Cinica Indica 4(2000) 389-392.

[11] Tapi U. D ,Thakur S S and Rathore G.P.S. Fuzzy sg –super continuous mappings Applied sciences periodical (2001) ,133-137.

[12] Tapi U. D., Thakur S. S. and Rathore G.P.S. Fuzzy semi generalized super closed sets, Acta Cien. Indica 27 (M) (3) (2001), 313-316.

[13] Tapi U. D., Thakur S. S. and Rathore G.P.S. Fuzzy sg- irresolute mappings stud. Cert. Stii. Ser. Mat. Univ. Bacu (1999) (9) ,203-209.

[14] Thakur S.S. & Malviya R., Generalized super closed sets in fuzzy topology Math. Note 38(1995),137-140.

[15] Yalvac T.H. fuzzy sets and Function on fuzzy spaces, J. Math. Anal. App. 126(1987), 409-423.

[16] Yalvac T.H. Semi interior and semi closure of fuzzy sets,. J. math. Anal.Appl. 132 (1988) 356-364.

[17] Zadeh L.A. fuzzy sets, Inform and control 18 (1965), 338-353.