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Four Force in MOND

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Abstract: *The transformations of a four force in MOND theory are deduced. They differ to their relativistic counter parts.*

I. Introduction

The modified Newtonian dynamics (MOND) is a burgeoning theory that proposes a modification of Newton's second law of dynamics $\mathbf{F} = m\mathbf{a}$ to explain the galaxy rotation problem.When the uniform velocity of rotation of cluster of galaxies was first observed (Oort 1932 and Zwicky 1933), it was not in tune with Newtonian theory of gravity- the galaxies or stars sufficiently away from the centre of the cluster move with constant velocities more than predicted by the theory. The dark matter was, then postulated to account for this constant large velocity. However, even after seven decades, there is not a convincing evidence of the dark matter. In an attempt to explain the observed uniform velocities of galaxies, Professor Milgrom in 1983 propounded an equation of motion that resulted into a theory which is known as MOND (modified Newtonian dynamics). The Newton's second law of motion now is generalized to

$$\mathbf{F} = m\,\mu\left(\frac{a}{a_0}\right)\mathbf{a}, \qquad (1.1)$$

which can be considered as modification of Newtonian dynamics,

where $\mu\left(\frac{a}{a_0}\right) = \begin{cases} a/a_0 & a << a_0 \\ 1 & a >> a_0 \end{cases}$.

For $a >> a_0$ above equation reduces to $\mathbf{F} = m\mathbf{a}$ which is Newtonian dynamics. The quantity a_0 is constant and has the dimension of acceleration and is evaluated as $a_0 \approx 2\times10^{-8} cm/s^2$ (Milgrom 1983a, Bekenstein and Milgrom 1984).Basically the early MOND is non-relativistic and hence will be interesting to probe into the consequences of MOND coupled with Lorentz symmetry. As a first step towards the goal, in this paper, we deduce the transformation of four force in the MOND region. It is observed that the transformations to that effect are different from their relativistic counter parts, as is expected.

II. The Relativistic Analogue Of MOND In Regard To Velocity

The Newton's second law of motion in the usual notation can be expressed as

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = \frac{d}{dt}(m\mathbf{u}),$$

In relativity when $\mathbf{u}$ is the velocity of a particle with mass $m = m(u)$, above gives

$$\mathbf{F} = m\frac{du}{dt} + \mathbf{u}\frac{dm}{dt} \qquad (2.1)$$

Comparing this equation with its analogue in MOND (1.1), we deduce

$$m\frac{a}{a_0} = m + u\frac{dm}{du}$$

Considering a to be a constant k, above linear differential equation reduces to a variable separable form

$$\frac{dm}{m} = \frac{du}{u}\left(\frac{k-a_0}{a_0}\right)$$

On integration, we obtain

$$\frac{m}{m_0} = \left(\frac{u}{u_0}\right)^A, \quad A = \left(\frac{k - a_0}{a_0}\right)$$

Noting $m = \dfrac{m_0}{\sqrt{1 - \dfrac{u^2}{c^2}}}$, above is rewritten as

$$\frac{m_0}{\sqrt{1 - \dfrac{u^2}{c^2}}} = m_0\left(\frac{u}{u_0}\right)^A \quad \text{or} \quad \left(1 - \frac{u^2}{c^2}\right)^{\frac{-1}{2A}} = \frac{u}{u_0}$$

Neglecting higher order terms of $\dfrac{u^2}{c^2}$, above yields

$$u^2 - \frac{2Ac^2}{u_0}u + 2Ac^2 = 0$$

For real values of u, we get

$$A \geq \frac{2u_0^2}{c^2} \text{ i.e. } \frac{k - a_0}{a_0} \geq 2\frac{u_0^2}{c^2}$$

or
$$k \geq a_0\left(1 + \frac{2u_0^2}{c^2}\right), \tag{2.2}$$

where we assume that $m(u_0) = m_0,\ u_0 \neq 0$.

This equation seems to be interesting and deserves physical interpretation. At present we conclude that the conservation of linear momentum is acclaimed for $k = 0$ which corresponds to a rectilinear motion of a particle. Also $m = m_0$ corresponds to $k = a_0$.

III. The relativistic analogue of MOND in regard to acceleration

From (1.1) and (2.1),

$$m\,\mathbf{a} + \frac{m\,\mathbf{u}(\mathbf{a}\cdot\mathbf{u})}{\beta^2 c^2} = m\mathbf{a}\mu, \tag{2.3}$$

where
$$m = \frac{m_0}{\beta} \text{ and } \beta = \sqrt{1 - \frac{u^2}{c^2}}.$$

Now we consider following two cases,

Case I: Let **a** be perpendicular to **u.** Then $\mathbf{a}\cdot\mathbf{u} = 0$ and $a = a_0$, follows from (2.3). Also (1.1) goes over to Newtonian dynamics.

Case II: Let $\mathbf{a}$ be parallel to $\mathbf{u}$. We consider $\mathbf{a} = B\mathbf{u}$, where B is a constant. We investigate the relativistic analogue of MOND in regard to velocity and acceleration as follows.

Equation (2.3) implies

$$\frac{m_0}{\beta} B\mathbf{u} + \frac{(m_0/c^2)(B\mathbf{u}\cdot\mathbf{u})}{\beta^3}\mathbf{u} = \frac{m_0}{\beta}\mu B\mathbf{u}.$$

This simplifies to

$$\left(\frac{a}{a_0}\right) = \frac{1}{\beta^2}. \tag{2.4}$$

4. Four force in MOND

Considering four momentum $p^i = (\mathbf{p}, p^4)$, we write

$$\frac{d\,\mathbf{p}}{ds} = \frac{m\mu(a/a_0)\mathbf{a}}{c\,\beta} \quad \text{or} \quad \frac{d\mathbf{p}}{ds} = \frac{m\mu\mathbf{a}}{c\beta}.$$

and
$$\frac{dp^4}{ds} = \frac{d}{dt}\left(\frac{E}{c}\right)\frac{dt}{ds} = \frac{dE}{dt}\frac{1}{c^2\beta} \tag{2.5}$$

where E represents energy.
The work done dw in moving a system through the displacement d$\mathbf{r}$ is

$$dw = \mathbf{F}\cdot d\mathbf{r} \ \text{ or } \ \frac{dw}{dt} = (m\mu\mathbf{a})\cdot\mathbf{u}$$

If the work done is used in increasing the kinetic energy T, above gives,

$$\frac{dw}{dt} = \frac{dT}{dt} = \mathbf{F}\cdot\mathbf{u}$$

$$\frac{dT}{dt} = \frac{dE}{dt} = \mathbf{F}\cdot\mathbf{u}$$

Using above equation, (2.5) becomes

$$\frac{dp^4}{ds} = \frac{(m\mu\mathbf{a})\cdot\mathbf{u}}{c^2\beta}.$$

Thus four force in MOND can be expressed by $F^i = \dfrac{dp^i}{ds}$ as a four force,

$$F^i = \left(\frac{m\mu\mathbf{a}}{c\beta}, \frac{(m\mu\mathbf{a})\cdot\mathbf{u}}{c^2\beta}\right) \tag{2.6}$$

IV. Transformations of the components of four force F^i in MOND

Considering its components F^1, F^2, F^3, F^4 must transform like the components of a four radius vector x^i under LTS i.e.

$$\left.\begin{aligned} F'^1 &= \alpha\left(F^1 - \frac{v}{c}F^4\right), \ \ \mathrm{F}'^2 = F^2, \ \ \mathrm{F}'^3 = F^3 \\ F'^4 &= \alpha\left(F^4 - \frac{v}{c}F^1\right), \ \ \alpha = \left(1 - \frac{\mathrm{v}^2}{\mathrm{c}^2}\right)^{-1/2} \end{aligned}\right\} \tag{2.7}$$

In view of (2.6) in S' frame, we write

$$F'^{i} = \left(\frac{m'\mu'\mathbf{a}'}{c\beta'}, \frac{(m'\mu'\mathbf{a}')\cdot\mathbf{u}'}{c^2\beta'} \right), \ \beta' = \sqrt{1-\frac{u'^2}{c^2}}$$

Substituting the values of F^1, F^2, F^3, F^4 from (2.6) in (2.7) and by using the result,

$$\sqrt{1-\frac{u'^2}{c^2}} = \frac{\sqrt{1-v^2/c^2}\sqrt{1-u^2/c^2}}{1-\frac{u_x v}{c^2}},$$

we get

$$m'\mu' a'_x = m\mu a_x - \frac{v\mu m}{(c^2-u_x v)}(a_y u_y + a_z u_z)$$

$$m'\mu' a'_y = \frac{c^2\sqrt{1-\frac{v^2}{c^2}}}{(c^2-u_x v)} m\mu a_y, \ \ m'\mu' a'_z = \frac{c^2\sqrt{1-\frac{v^2}{c^2}}}{(c^2-u_x v)} m\mu a_z$$

$$m'\mu'(\mathbf{a}'\cdot\mathbf{u}') = \frac{c^2 m\mu}{(c^2-u_x v)}[a_x(u_x - v) + (a_y u_y + a_z u_z)]$$

Using $a = |\mathbf{a}| \approx \frac{\sqrt{GMa_0}}{r}$, (see Milgrom (1983), Bekenstein (2004)), we get transformations of acceleration components

$$\frac{m'a'_x}{r'} = \frac{ma_x}{r} - \frac{mv}{r(c^2-u_x v)}(a_y u_y + a_z u_z),$$

$$\frac{m'a'_y}{r'} = \frac{c^2\sqrt{1-\frac{v^2}{c^2}}}{(c^2-u_x v)}\frac{ma_y}{r}, \ \ \frac{m'a'_z}{r'} = \frac{c^2\sqrt{1-\frac{v^2}{c^2}}}{(c^2-u_x v)}\frac{ma_z}{r},$$

$$m'\frac{(\mathbf{a}'\cdot\mathbf{u}')}{r'} = \frac{mc^2}{r(c^2-u_x v)}[a_x(u_x - v) + (a_y u_y + a_z u_z)$$

Defining $\mathbf{F}^{(N)} = m\mathbf{a}$ analogous to Newtonian force mass acceleration relationship, the above transformation can be rewritten as

$$\frac{F_x'^{(N)}}{r'} = \frac{F_x^{(N)}}{r} - \frac{v}{r(c^2-u_x v)}(F_y^{(N)} u_y + F_z^{(N)} u_z) \tag{2.8}$$

$$\frac{F_y'^{(N)}}{r'} = \frac{c^2\sqrt{1-\frac{v^2}{c^2}}}{c^2-u_x v}\frac{F_y^{(N)}}{r} \tag{2.9}$$

$$\frac{F_z'^{(N)}}{r'} = \frac{c^2\sqrt{1-\frac{v^2}{c^2}}}{c^2-u_x v}\frac{F_z^{(N)}}{r} \tag{2.10}$$

and
$$\frac{F_x^{'(N)}u'_x + F_y^{'(N)}u'_y + F_z^{'(N)}u'_z}{r'}$$
$$= \frac{c^2}{r(c^2 - u_x v)}[F_x^{(N)}(u_x - v) + F_y^{(N)}u_y + F_z^{(N)}u_z]. \qquad (2.11)$$

Conclusion

When $\mathbf{a}$ is perpendicular to $\mathbf{u}$, MOND boils down to Newtonian mechanics and for $\mathbf{a}$ parallel to $\mathbf{u}$ yields to (2.4), the relativistic analogue of MOND in regard to velocity and acceleration. The four force transformations in MOND are given by the equations (2.6), (2.8), (2.9), (2.10) and (2.11).

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References

[1.] Oort J, Bull. Astron. Inst. Neth. **6,** 249 1932
[2.] Zwicky F, Helv. Phys. Acta **6,** 110 1933
[3.] Milgrom M, 1983a, Ap J, **270**, 365
[4.] Milgrom M, 1983b, Ap J, **270**, 371.
[5.] Milgrom M, 1983c, Ap J, **270**, 384.
[6.] Bekenstein J, 2004, Phys Rev D, **70**, 083509.