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Fuzzy Game Value by Dominance Principle

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Abstract: *When there is no saddle interval, applying the dominance principle, we determine the value of the game for the interval valued fuzzy game matrix by reducing the size of the given matrix.*

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I. Introduction

Payoffs in games are usually not known precisely, but it is often possible to determine lower and upper bounds on payoffs .Hence we consider interval valued fuzzy matrices. In [4], method of finding the fuzzy game value is explained. Definitions of intervals and interval arithmetic are studied in [1].By comparing the intervals and using dominance principle [2] in a fuzzy environment we find the fuzzy game value of the interval valued matrix.

II. Interval Arithmetic

2.1 Definition:
The extension of ordinary arithmetic to intervals is known as interval arithmetic.
Let A = $[a_L, a_R]$ and B = $[b_L, b_R]$ be two intervals.
Then,

(i) A + B = $[a_L + b_L,\ a_R + b_R]$.

(ii) A – B = $[a_L - b_R,\ a_R - b_L]$.

(iii) AB = $[\min\{a_L b_L, a_L b_R, a_R b_L, a_R b_R\}, \max\{a_L b_L, a_L b_R, a_R b_L, a_R b_R\}]$

(iv) λ A = $[\lambda a_L, \lambda a_R]$, if $\lambda \geq 0$ and $= [\lambda a_R, \lambda a_L]$, if $\lambda < 0$.

Similarly the other binary operations are defined

2.1Coimparison of Intervals:
Comparison of two intervals is very important problem in interval analysis. In this section we consider the order relation (≤ and ≥) between intervals.

2.2 Disjoint Intervals:
If a = $[a_L, a_R]$, b = $[b_L, b_R]$ and if $a_R < b_L$ then a < b or b > a crisply, which is similar to the definition of comparisons used in [4]

2.3 Equal Intervals:
a = b iff a ≤ b and a ≥ b.

2.5: Overlapping Intervals:
If $a_L < b_L < a_R < b_R$ then for any x in $[a_L, b_L]$, a< b. That is x∈ a, is less than every payoff in b.
If x $\in [b_L, a_R]$, then every x ∈ a, is less than or equal to b. Therefore a ≤ b (crisply). Similarly if y ∈ $[a_R, b_R]$, then y in b is greater than or equal to a. That is b ≥ a crisply.

III. Nested sub intervals

In terms of fuzzy membership

$$A \leq b = \frac{b_{R-}a_R}{b_R - b_L}$$

If $b \subseteq a$ then

$$A \leq b = \frac{b_L - a_L}{a_R - a_L}$$

Consolidating the above discussions we define the fuzzy operations $\prec$ *and* $\succ$ as follows.

3.1 Definition:

The binary fuzzy operator $\prec$ of two intervals a and b returns a real number between 0 and 1 as follows

$a \prec b$ = 1 if a = b or $a_R < b_L$, a ≠ b; or $a_L < b_L < a_R < b_R$.

0 if $b_R < a_L$, $b_L < a_L < b_R < a_R$.

$\frac{b_L - a_L}{a_R - a_L}$ If $a_L < b_L < b_R < a_R$.

$\frac{b_{R-}a_R}{b_R - b_L}$ if $b_L < a_L < a_R < b_R$.

These values $\frac{b_L - a_L}{a_R - a_L}$ and $\frac{b_{R-}a_R}{b_R - b_L}$ moves from 0 to 1 when x moves a_L to b_L and a_R to b_R from left to right. Only when a=b, $a \prec b$ takes the value 1.Using simple algebraic operations, it can be seen that the membership value for $b \succ a = 1 - a \prec b$.

3.2 Definition:

The binary fuzzy operator ≥ of two intervals a and b is defined as

a ≥ b= 1 if a = b ;or $b_R < a_L$ or $b_L < a_L < b_R < a_R$.

0 if $a_R < b_L$ a ≠ b, $a_L < b_L < a_R < b_R$

$\frac{a_R - b_R}{a_R - a_L}$ if $a_L < b_L < b_R < a_R$.

$\frac{a_L - b_L}{b_R - b_L}$ if $b_L < a_L < a_R < b_R$.

The relations of two intervals can now be either crisp or fuzzy as described below.

3.3 Definition:

If the values of a ≤ b is exactly one or zero then we say that a and b are crisply related otherwise we say that they are fuzzily related.

IV. Crisp Game value of the Matrix:

The ideas and concept of crisp game value of the matrixes can be extended to the matrix games with interval data where entries are crisply related.

4.1 Definition:

Let A be an mxn interval game matrix such that all the intervals in the same row (or column) of A are crisply related. If there exists a $g_{ij} \in A \ni a_{ij}$ is simultaneously crisply less than or equal to $a_{ik} \forall k = 1.........n$ and crisply $\geq g_{lj} \forall l = 1...m$, then the interval g_{ij} is called a saddle interval of the game. The value of the saddle interval is called the crisp game value of interval matrix game

V. Fuzzy game value of the interval matrix

The crisp relativity may not be satisfied for all intervals in the same row (or column) .We now define the fuzzy memberships of an interval being a minimum and a maximum of an interval vector R and then we define the notion of a least and greatest interval in R.

5.1 Definition:

The least interval of the vector R is defined as

$$\max_{1\leq i \leq n,\ 1 \leq j \leq m} \{\min \{ r_i \leq r_j \}\} = l$$

Similarly the greatest interval of the vector R is defined as

$$\min_{1\leq j \leq m,\ 1 \leq i \leq n} \{\max \{ r_i \geq r_j \}\} = g$$

5.2 Definition:

Let R be an mxn interval game matrix. If there is an $r_{ij} \in R \ni r_{ij}$ is simultaneously the least and the greatest interval for the ith row and jth column of R, then that interval value is the fuzzy game value of the interval valued matrix game.

VI. Dominance Principle

6.1 Definition:

Let A = [a_L, a_R], B = [b_L, b_R] be two intervals. Then we say that A or b is dominated by B (or A) in the sense of minimization (or maximization) if A $\prec$ B or B $\prec$ A.

6.2 General Rules for Dominance;

(a) If all the elements of the ith row are less than or equal to the corresponding elements of any other row say rth row then ith row is dominated by the rth row.
(b) If all the elements of jth column are greater than or equal to the corresponding elements of any other column say kth column, then jth column is dominated by the kth column.
(c) Dominated columns or rows may be deleted to reduce the size of the payoff matrix as optimal strategies will remain unaffected.
(d) A given strategy can also said to be dominated if it is inferior to an average of two or more other pure strategies. More generally if some convex linear combination of some rows dominates the i th row, then i th row will be deleted. Similar arguments follow and columns.
(e) Thus the given matrix can be reduced to a simple matrix for which the fuzzy game value can be evaluated easily.

6.3 Example:

Consider the matrix

$$\begin{bmatrix} [2,4] & [0,1] & [-4,-1] & [1,2] \\ [5,6] & [3,5] & [-1,2] & [4,5] \\ [-3,0] & [-2,-3] & [-5,-3] & [5,6] \end{bmatrix}$$

(i) First row elements are less than the second row elements. Therefore the first row is dominated by the other. Then the first row is deleted. Hence we have,
(ii)

$$\begin{bmatrix} [5,6] & [3,5] & [-1,2] & [4,5] \\ [-3,0] & [-2,-3] & [-5,-3] & [5,6] \end{bmatrix}$$

(ii) First column elements are greater than the second column elements. Therefore the first column is dominated by the other. Then the first column is deleted. Hence we have,

$$\begin{bmatrix} [3,5] & [-1,2] & [4,5] \\ [-2,-3] & [-5,-3] & [5,6] \end{bmatrix}$$

(iii) Third column elements are greater than the first column elements. Therefore the third column is dominated by the other. Then the third column is deleted. Hence we have,

$$\begin{bmatrix} [3,5] & [-1,2] \\ [-2,-3] & [-5,-3] \end{bmatrix}$$

In this matrix the saddle interval is given by [-1, 2].This is the game value for the given matrix.

Conclusion:

By using the interval arithmetic, comparison of intervals and dominance principle we can always reduce the size of the matrix. Either the crisp value of the game or the fuzzy game value can be evaluated for the reduced matrix. This can be extended to a multi player game also.

References:

[1] Loganathan.C and Annie Christi.M.S, Fuzzy Game Value of the Interval Matrix, International Journal of Engineering Research and Applications, Vol: 2, Issue 5, (2012).

[2] Madhumangal Pal and Prasun K. Nayak, Solutions of m × n Rectangular Fuzzy Games by dominance, (2004).

[3] Porchelvi.T and Stephen Dinagar D, Fuzzy Dominance Principle, International journal of Algorithms, Computing and Mathematics, Volume 3, (2010).

[4] Van Neumann and Morgenstern.O, Theory of games and Economic Behaviour, Princeton. N.J. Princeton University press, (1944).