

Designing Pd Controller for a Parallel Manipulators

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Abstract: Parallel robots have received increasing attention due to their inherent advantages over conventional serial mechanism, such as high rigidity, high load capacity, high velocity, and high precision. A definite advantage of parallel robots is the fact that, in most cases, actuators can be placed on the truss, thus achieving a limited weight for the moving parts, which makes it possible for parallel robots to move at a high speed. These advantages avoid the drawbacks on serial ones and make the mobile platforms of the parallel manipulators carry out higher performances. This paper using the PD controler to control parallel manipulator based on mechanical models of parallel manipulators.

Keywords: Parallel manipulator, DAE, PD control.

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I. INTRODUCTION

The equations of motion for a multibody system are obtained as the end result of a sequence of mathematical operators. In general, the known methods to derive the equations of motion of multibody systems are Lagrange's equations, Newton–Euler equations, Kane's equations. Among these methods, the approach using Lagrange's equations with multipliers has become an attractive method to derive the equations of motion of constrained multibody systems. This approach provides a well analytical and orderly structure that is very useful for control purposes.

The control of treelike multibody systems is of interest to a number of research communities in a very of applications areas. Many advanced methods for control of robot manipulators based on the Lagrange's equations have been developed [1-11]. The application of modern control methods such as Sliding Mode Control method, the neural network control method for controller design of the treelike robot manipulators is presented in the works [12-21]. In the present study, we present a control method for controller design of spatial parallel robot manipulator, the application of the new matrix form of Lagrangian equations with multipliers for constrained multibody systems.

II. KINEMATIC MODEL

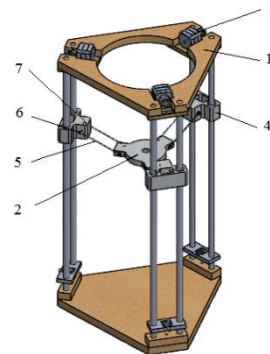


Fig. 1. Model of a Delta 3-PRS manipulator

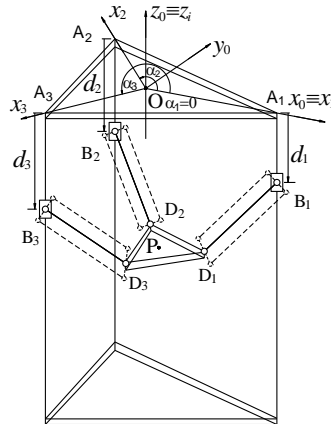
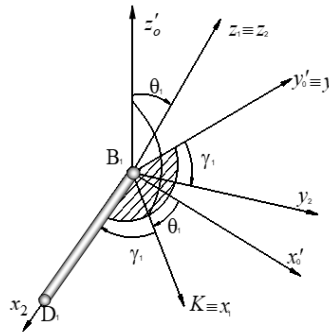


Fig. 2. Robot kinematic diagram


 Fig. 3. Positioning diagram of B_iD_i in the space

For the sake of simplicity in system dynamics analysis, parallelogram stages are modeled by bars B_1D_1 , B_2D_2 , and B_3D_3 . These bars of length l are connected to the stitches driven by Cardan joints and connected to the machine table by ball joints. These bars of length l are connected to the sliding driven stage by Cardan joints and connected to the moving table by ball joints. B_iD_i stages are positioned by angles θ_i , γ_i , as shown in Fig. 3. For convenience, the following symbols are included :

$$\mathbf{q}_a = [d_1 \quad d_2 \quad d_3]^T, \mathbf{q}_p = [\theta_1 \quad \theta_2 \quad \theta_3 \quad \gamma_1 \quad \gamma_2 \quad \gamma_3]^T, \mathbf{x} = [x_p \quad y_p \quad z_p]^T \quad (1)$$

where $\mathbf{q}_a, \mathbf{q}_p, \mathbf{x}$ are the independent generalized coordinates of active joints, passive joints, and coordinates of the center P of the moving table B (manipulation coordinates). Thus, the generalized coordinates are $\mathbf{s} = [\mathbf{q}_a^T \quad \mathbf{q}_p^T \quad \mathbf{x}^T]^T$.

In this model, the parallelogram joints are replaced by solid bars whose mass is evenly distributed over the bar length. The weight and length of the bar are equal to the weight and length of the parallelogram joint.

$$\mathbf{s} = [d_1 \quad d_2 \quad d_3 \quad x_p \quad y_p \quad z_p]^T \in \mathbb{R}^6 \quad (2)$$

Applying the Lagrange factor equation, we can establish the motion equation of the robot in the form of a matrix as below

$$\mathbf{M}(\mathbf{s})\ddot{\mathbf{s}} + \mathbf{C}(\mathbf{s}, \dot{\mathbf{s}})\dot{\mathbf{s}} + \mathbf{g}(\mathbf{s}) + \Phi_s^T(\mathbf{s})\boldsymbol{\lambda} = \boldsymbol{\tau} \quad (3)$$

$$\mathbf{f}(\mathbf{s}) = \mathbf{0} \quad (4)$$

where:

$$\boldsymbol{\tau} = [F_1 \quad F_2 \quad F_3 \quad 0 \quad 0 \quad 0]^T; \boldsymbol{\lambda} = [\lambda_1 \quad \lambda_2 \quad \lambda_3]^T; \Phi_s(\mathbf{s}) = \frac{\partial \mathbf{f}}{\partial \mathbf{s}} \in \mathbb{R}^{3 \times 6}$$

$$\mathbf{M}(\mathbf{s}) = \text{diag} \left[\begin{array}{ccc} m_1 + \frac{1}{2}m_2, & m_1 + \frac{1}{2}m_2, & m_1 + \frac{1}{2}m_2, \\ m_p + \frac{3}{2}m_2, & m_p + \frac{3}{2}m_2, & m_p + \frac{3}{2}m_2 \end{array} \right]$$

$$\mathbf{g}(\mathbf{s}) = - \left[\left(m_1 + \frac{1}{2}m_2 \right) g \quad \left(m_1 + \frac{1}{2}m_2 \right) g \quad \left(m_1 + \frac{1}{2}m_2 \right) g \quad 0 \quad 0 \quad \left(m_p + \frac{3}{2}m_2 \right) g \right]^T$$

The constraint equations are established based on the following conditions:

$$(x_{Bi} - x_{Ei})^2 + (y_{Bi} - y_{Ei})^2 + (z_{Bi} - z_{Ei})^2 - l^2 = 0$$

Therefore, the constraint equations have the following form:

$$f_i = l^2 - (\cos \alpha_i (R - r) - x_p)^2 - (\sin \alpha_i (R - r) - y_p)^2 - (d_i + z_p)^2 = 0 \quad \text{with } i=1, 2, 3$$

The matrix $\mathbf{C}(\mathbf{s}, \dot{\mathbf{s}})$ is calculated from the mass matrix $\mathbf{M}(\mathbf{s})$ using the Kronecker product as follows:

$$\mathbf{C}(\mathbf{s}, \dot{\mathbf{s}}) = \frac{\partial \mathbf{M}(\mathbf{s})}{\partial \mathbf{s}} (\mathbf{E} \otimes \dot{\mathbf{s}}) - \frac{1}{2} \left(\frac{\partial \mathbf{M}(\mathbf{s})}{\partial \mathbf{s}} (\dot{\mathbf{s}} \otimes \mathbf{E}) \right)^T \quad (5)$$

where \mathbf{E} is a unit matrix of the size of the vector \mathbf{s} . Since $\mathbf{M}(\mathbf{s})$ is a diagonal matrix whose elements are constants, $\mathbf{C}(\mathbf{s}, \dot{\mathbf{s}}) = \mathbf{0}$.

III. CONTROL DESIGN

From the constraint equation $\mathbf{f}(\mathbf{s}) = \mathbf{0}$, solving the inverse kinematics problem, we obtain the desired position, velocity, and acceleration of the active joints of the robot $\mathbf{q}_a^d(t)$, $\dot{\mathbf{q}}_a^d(t)$, $\ddot{\mathbf{q}}_a^d(t)$. Denoting $\mathbf{q}_a(t)$, $\dot{\mathbf{q}}_a(t)$, $\ddot{\mathbf{q}}_a(t)$ as the actual position, velocity, and acceleration of the active joints of the robot. Then, the positional error has the following form :

$$\mathbf{e}(t) = \mathbf{q}_a(t) - \mathbf{q}_a^d(t) \quad (6)$$

We choose the control rule $\mathbf{u}(t)$ as below :

$$\mathbf{e}(t) = \mathbf{q}_a(t) - \mathbf{q}_a^d(t) \rightarrow \mathbf{0} \quad (7)$$

with

$$\mathbf{v} = \ddot{\mathbf{q}}_a^d - \mathbf{K}_D \dot{\mathbf{e}} - \mathbf{K}_P \mathbf{e} \quad (8)$$

where \mathbf{K}_D , \mathbf{K}_P are positive definite diagonal matrices:

$$\dot{V}(t) = \mathbf{n}^T \bar{\mathbf{M}} \dot{\mathbf{n}} + \frac{1}{2} \mathbf{n}^T \bar{\mathbf{M}} \dot{\mathbf{n}} + \sum_{i=1}^{n_a} \mathbf{w}_i^T \mathbf{w}_i, \quad k_{P_i} > 0, \quad k_{D_i} > 0$$

We get:

$$\begin{aligned} \bar{\mathbf{M}} (\ddot{\mathbf{q}}_a^d - \mathbf{K}_D \dot{\mathbf{e}} - \mathbf{K}_P \mathbf{e}) + \bar{\mathbf{C}} \dot{\mathbf{q}}_a + \bar{\mathbf{g}} \\ = \bar{\mathbf{M}} \ddot{\mathbf{q}}_a + \bar{\mathbf{C}} \dot{\mathbf{q}}_a + \bar{\mathbf{g}} \end{aligned} \quad (9)$$

From (9) we have:

$$\bar{\mathbf{M}} [\ddot{\mathbf{e}} + \mathbf{K}_D \dot{\mathbf{e}} + \mathbf{K}_P \mathbf{e}] = \mathbf{0} \quad (10)$$

Since $\bar{\mathbf{M}}$ is a positive deterministic matrix, from (10) we have:

$$\ddot{\mathbf{e}} + \mathbf{K}_D \dot{\mathbf{e}} + \mathbf{K}_P \mathbf{e} = \mathbf{0} \quad (11)$$

Since \mathbf{K}_D , \mathbf{K}_P are two diagonal matrices, from (11) we receive:

$$\ddot{e}_i + k_{D_i} \dot{e}_i + k_{P_i} e_i = 0 \quad (i = 1, 2, \dots, n_a) \quad (12)$$

Define:

$$k_{P_i} = \omega_i^2, \quad k_{D_i} = 2\delta_i \quad (13)$$

Then, solution of (12) has the following form:

$$e_i(t) = A_i e^{-\delta_i t} \sin(\omega_i t + \alpha_i) \rightarrow 0 \quad \text{when } t \rightarrow \infty \quad (14)$$

If \mathbf{K}_D , \mathbf{K}_P are chosen as diagonal matrices with positive elements, the system of differential equations (12) will be exponentially stable. The control technique is straightforward. However, it is necessary to know precisely the matrices and vectors $\bar{\mathbf{M}}_a$, $\bar{\mathbf{C}}_a$, $\bar{\mathbf{g}}_a$, $\bar{\mathbf{d}}_a$, that is, to know precisely the model parameters. In other words, accurate model parameters are required.

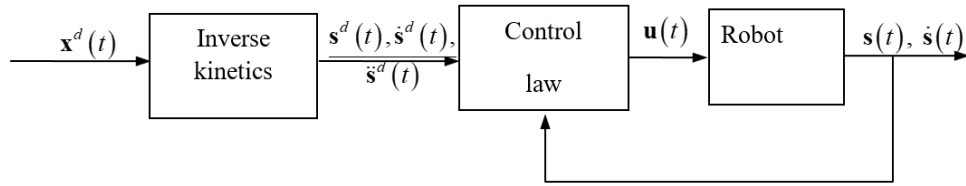


Fig. 4. Control diagram

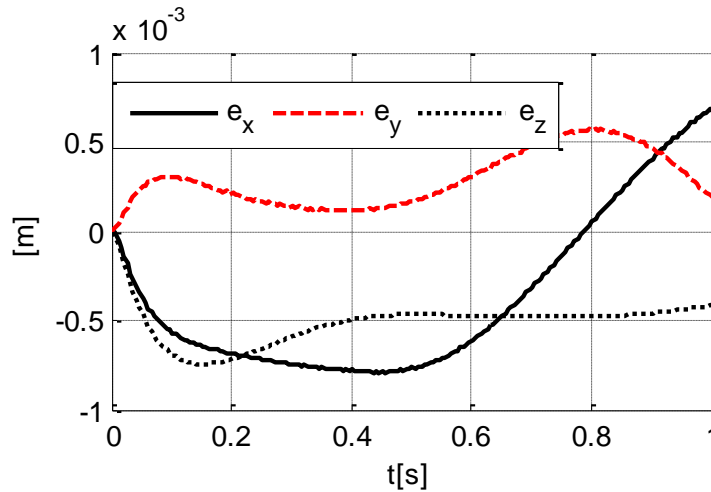


Fig. 5. Position error

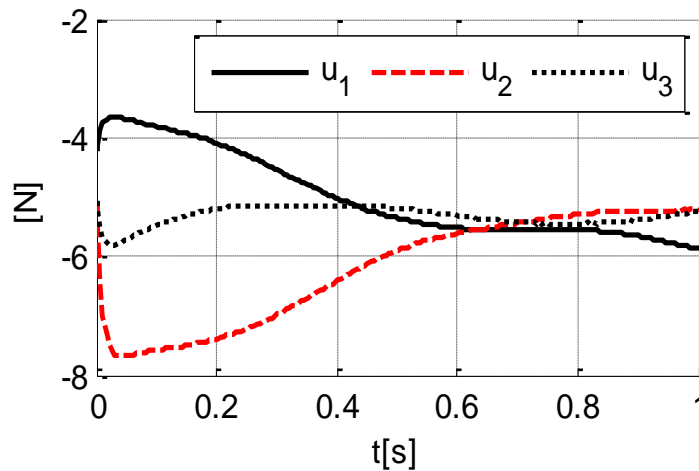


Fig. 6. Torque control

IV. CONCLUSION

This paper presents the construction of controllers for parallel manipulators based on a system of algebraic differential equations. The application of the PD control law to compensate uncertainties in the parallel robot manipulators. The new matrix form of Lagrangian equations with multipliers for constrained multibody systems was used to derive dynamic equations of spatial parallel robot manipulators. The simulation results showed PD control method good results when using enough accurate mechanical models.

Conflict of interest

There is no conflict to disclose.

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