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Simple Approximate Estimate of the Heat Transfer Characteristics of Annular Fins of Hyperbolic Profile with Combined Integral Method

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Abstract:
It is undeniable that the annular fin of hyperbolic profile with constant thermal conductivity and uniform convective coefficient is important in many applications of heat transfer engineering. The importance of this fin configuration stems from its close resemblance to the annular fin of optimal cross section capable of delivering maximum heat transfer for a given volume of material. In the present paper, a new combined integral method, based on two integral relations, is presented for approximate solving the generalized Bessel equation defining the change in the temperature of hyperbolic-profile annular fins. Certainly, the central objective here is to avoid the evaluation of the elegant, but intricate exact analytic temperature distributions and companion fin efficiencies containing modified Bessel functions of fractional order. Results are presented for the two variables of interest in thermal design: the fin tip temperature (a local quantity) and the fin thermal efficiency (a global quality ratio).

Keywords: *Heat transfer, hyperbolic annular fin, fin efficiency.*

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I. INTRODUCTION

In modern era, heat exchange devices are becoming increasingly sophisticated and continually require greater precision, adequate sizing, improved reliability, and extended life [1–5]. To meet these stringent demands, exceptional fin profiles have been ceaseless explored in theoretical studies, numerical simulations and experimental measurements all over the world [1–5]. From geometrical considerations, the annular fin can have six different profiles: uniform, triangular, trapezoidal, concaveparabolic, convex parabolic and hyperbolic. A review of heattransfer textbooks written over the last 30 years reveals that the explanation of annular fins is solely restricted to the annular finof uniform profile. While most textbooks resort to the simplisticfin efficiency diagram exclusively [6-18], someothers [19–24] formulate the governing quasi 1-Dfin equation, deduce the temperature distribution along with theheat transfer rate through the fin efficiency. In this regard, the annular fin of hyperbolic profile turns out to be the foremost important fin that can be attached to round tubes because it resembles the optimal annular fin of convex parabolic profile discovered by Schmidt [25]. Unquestionably, the latter has become staple in heat transfer engineering because of its unique ability to reject maximum heat transfer for a given volume of metallic material [1–5]. From a fundamental standpoint, the temperature change along an annular fin of hyperbolic profile with constant thermal conductivity and uniform convective coefficient is governed by a two-term differential equation of second-order with a variable coefficient. By virtue of a proper transformation, the differential equation falls under the category of a generalized Bessel equation. Although this equation admits an exact analytical solution, it is of intricate form because of the presence of modified Bessel functions of fractional order. Hence, the numerical evaluation of temperatures and/or heat transfer rates is quite complicated and time-consuming.

Among the existing family of annular fins possessing tapered cross sections, it is widely recognized that the annular fin of hyperbolic profile is the foremost fin shape candidate for practical applications [1–3]. From an optimization standpoint, the annular fin of hyperbolic profile closely resembles the optimal annular fin of convex parabolic profile. As far as the modeling is concerned, the temperature change along an annular fin of hyperbolic profile is governed by a quasi-onedimensional heat equation, the socalled generalized Bessel equation. Despite that this equation admits an analytical solution for combinations of the enlarged Biot number M^2 and the normalized radii ratio c, the evaluation of local temperatures and heat transfer rates with modified Bessel functions of first kind and fractional order is complicated and time-consuming. Kraus et al. [2] present an analytical solution for a convecting annular hyperbolic fin of constant thermal conductivity in terms of modified Bessel functions. Inrecentyears, Arauzo et al. [26] analyzed the problem considered by Krausetal. [2] and derived a truncated series solution and claimed the solution. In another paper, Campoand Cui [27] applied a coordinate transformation to convert the Bessel equation into a differential equation for a straight fin which admits analytical solution in terms of the hyperbolic functions. More recently, analogous investigations nave been carried out by Campo and Lira [28]. Campoetal. [29] have performed a heat calculation of a hyperbolic-profilefin with the use of two simple numerical methods of solving the generalized Bessel equation governing the temperature variation in hyperbolic-profileannular fins, one of which represents a finite-difference technique with an uncharacteristic coarse mesh, and the other is a shooting technique . Certainly, the central objective here is to avoid the evaluation of the elegant, but intricate exact analytic temperature distributions and companion fin efficiencies containingmodified Bessel functions of fractional order. Recently, Yangetal. [30] used adouble decomposition method to analyze anannular fin of hyperbolic profile with temperature dependent (linear) thermal conductivity.

The present paper addresses the combined integral method as an alternate computational procedure for solving the governing quasi-one-dimensional heat equation in approximate manner. Due to its inherent simplicity, the combined integral method may be attractive to thermal design engineers and also to instructors of graduate courses on heat transfer. Factors influencing the structure of the power series solutions and their exactness will be discussed at length.

II. MATHEMATICAL MODEL

2.1. Formulation

An annular fin of hyperbolic profile is formed with the path oftwo symmetric hyperbolas $y(r)=\delta_1(r_1/r)$, as displayed in Fig. 1. The sizing of this fin depends on four dimensions: the inner radius r_1, the inner semithickness δ_1, the outer radius r_2, and the outer semithickness δ_2. According to [4], the cross section of the annular fin of hyperbolic profile is of remarkable significance becauseof its affinity to the cross section of the annular fin of convex parabolic profile capable of delivering maximum heat transfer for a given volume of material. In this sense, the latter fin has been aptly named the optimal annular fin in [31].

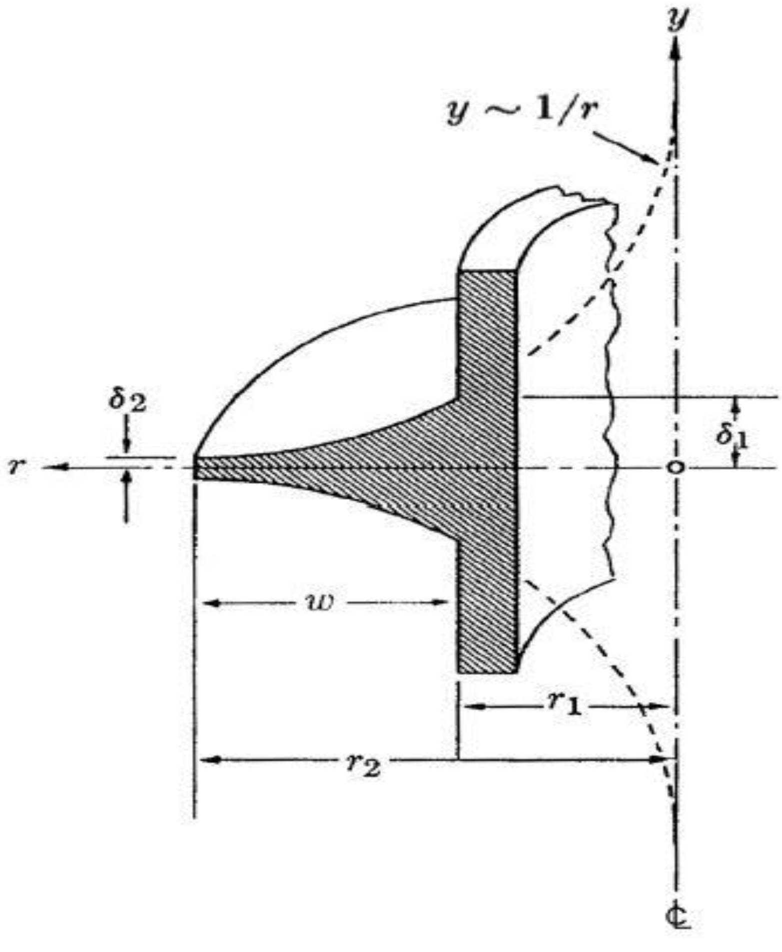


Figure 1. Sketch of an annular fin of hyperbolic profile

The temperature variation along the annular fin of hyperbolic profile obeys the dimensionless fin equation [31]:

$$R^2 \frac{\mathrm{d}^2\theta}{\mathrm{d}R^2} - M^2 R^3 \theta = 0 \tag{1}$$

subject to the boundary conditions of prescribed temperature at the fin base

$$\theta = 1, \qquad R = c \tag{2}$$

and negligible heat loss at the fin tip

$$\frac{\mathrm{d}\theta}{\mathrm{d}R} = 0, \qquad R = 1 \tag{3}$$

The dimensionless variables for the temperature h and the radial coordinate R used in Eqs. (1) and (2) are

$$\theta = \frac{T - T_\infty}{T_b - T_\infty}, \qquad R = \frac{r}{r_2} \tag{4}$$

The two parameters that surface up in the formulation are the enlarged Biot number $M^2 = hr_2^3 / (k\delta_1 r_1)$ in Eq. (1) and the normalized radii ratio $0 < c = r_1 / r_2 \leq 1$.

The trademark of the class of quasi-1D fin equations descriptiveof annular fins in cylindrical coordinates alludes to the curvature term $\mathrm{d}\theta / \mathrm{d}R$ [1–4, 30]. Nonetheless, the absence of $\mathrm{d}\theta / \mathrm{d}R$ in thequasi-1D fin equation (2) for the annular fin of hyperbolic profileis striking. The heat transfer rate Q from a fin to a neighboring fluid is customarily computed indirectly with the fin efficiency

$$\eta = \frac{Q}{Q_{\text{ideal}}} \tag{5}$$

in two equivalent ways: utilizing the derivative of $\theta(R)$ at the fin base

$$\eta = \frac{-2\left(\frac{\mathrm{d}\theta}{\mathrm{dR}}\right)_{R=c}}{M^2(1-c^2)} \tag{6}$$

or employing the integral of $\theta(R)$ over the fin length

$$\eta = \frac{2\int_c^1 \theta R \mathrm{d}R}{1-c^2} \tag{7}$$

2.2. Exact analytic solution

The general solution of the generalized Airy equation (2) is [32]

$$\theta(\mathrm{R}) = C_1 \,\mathrm{Ai}(M^{2/3}R) + C_2 \,\mathrm{Bi}(M^{2/3}R) \tag{8}$$

where $\mathrm{Ai}(*)$ and $\mathrm{Bi}(*)$ are the Airy functions [32]. For the special case, the alternative general solution is

$$\theta(R) = \sqrt{\frac{R}{c}} \frac{I_{1/3}\left(\frac{2m}{3\sqrt{c}} R^{3/2}\right) I_{2/3}\left(\frac{2m}{3\sqrt{c}}\right) - I_{-1/3}\left(\frac{2m}{3\sqrt{c}} R^{3/2}\right) I_{-2/3}\left(\frac{2m}{3\sqrt{c}}\right)}{I_{1/3}\left(\frac{2m}{3} c\right) I_{2/3}\left(\frac{2m}{3\sqrt{c}}\right) - I_{-1/3}\left(\frac{2m}{3} c\right) I_{-2/3}\left(\frac{2m}{3\sqrt{c}}\right)} \tag{9}$$

where $I_v(*)$ denotes the modified Bessel functions of first kind offractional order v, $m^2 = M^2 / c = h r_2^2 / (\delta_1 k c^2)$. Similarly, the exact analytic fin efficientcy η^* is

$$\eta^* = \frac{2c^{3/2}}{m(1-c^2)} \frac{I_{-2/3}\left(\frac{2m}{3} c\right) I_{2/3}\left(\frac{2m}{3\sqrt{c}}\right) - I_{2/3}\left(\frac{2m}{3} c\right) I_{-2/3}\left(\frac{2m}{3\sqrt{c}}\right)}{I_{-1/3}\left(\frac{2m}{3} c\right) I_{-2/3}\left(\frac{2m}{3\sqrt{c}}\right) - I_{1/3}\left(\frac{2m}{3} c\right) I_{2/3}\left(\frac{2m}{3\sqrt{c}}\right)} \tag{10}$$

III. APPROXIMATE ANALYTICAL METHODS

3.1. The Mean Value Theorem for Integration [26, 27]

Let us first isolate the troublesome dimensionless variable coefficient R in Eq. (2). Then, R can be conceived as a function outlining a straight line from the base $R = c$ to the tip $R = 1$ in the closed interval [c,1]. Upon applying the mean value theorem for integration to the function R, the result is

$$\bar{R} = \frac{1}{1-c} \int_c^1 R \,\mathrm{d}R = \frac{1+c}{2} \tag{11}$$

where $\bar{R}$ denotes the functional mean of R^2. The idea now is to replace R by $\bar{R}$ in Eq. (2), so that the dimensionless quasi-1D fin equation is converted to

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}X^2} - M^2\bar{R}\theta = 0 \tag{12}$$

As opposed to Eq. (2) that contains one variable coefficient, the product M^2R, now Eq. (9) possesses one constant coefficient, the product $M^2\bar{R}$. For conciseness, let us name a new constant coefficient $W^2 = M^2\bar{R}$ and concurrently introduce the coordinate transformation $X = R - c$. The quasi-1D fin equation (9) evolves into

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}X^2} - W^2\theta = 0, \qquad 0 \le X \le 1-c \tag{13}$$

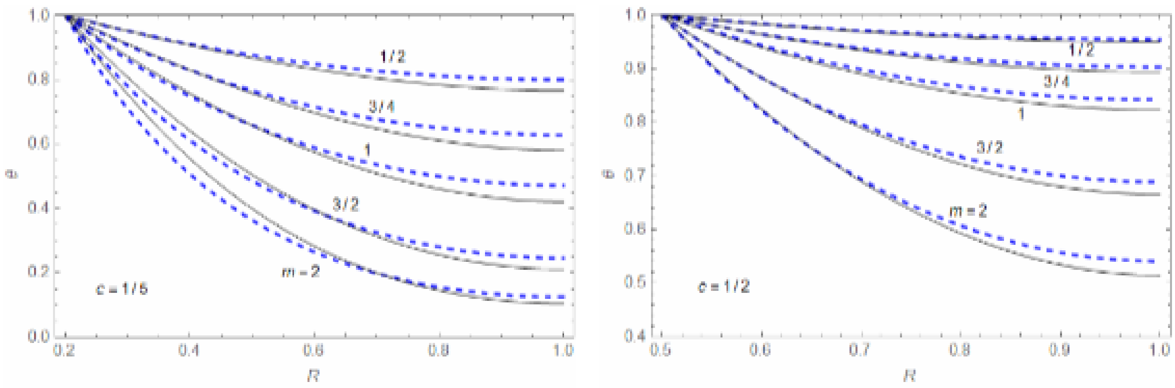
so that the two boundary conditions in (3) are

$$\theta(0) = 1, \qquad \frac{\mathrm{d}\theta(1-c)}{\mathrm{d}X} = 0 \tag{14}$$

At this juncture, it can be asserted that the new formulation given by Eqs. (11) and (12) is identical to the formulation for a straight fin of uniform profile of length $(1-c)$ [4, 6]. Skipping the algebra, the solution of Eqs. (11) and (12) when rewritten in terms of R, renders the approximate temperature distribution of form

$$\theta(R) = \frac{e^{WR} + e^{W(2-R)}}{e^{Wc} + e^{W(2-c)}} = \frac{e^{m\sqrt{\frac{1+c}{2c}}R} + e^{m\sqrt{\frac{1+c}{2c}}(2-R)}}{e^{m\sqrt{\frac{(1+c)c}{2}}} + e^{m\sqrt{\frac{1+c}{2c}}(2-c)}} \tag{15}$$

which embraces the two original parameters c and m. In Figure 2, wehave plotted the temperature distributions in a fin with $c = 1/5$ (a) and $c = 1/2$ (b) for parameter m ranging from $1/2$ to 2. We state a fairly coarse approximate solution for the function $\theta(R)$ obtained on the basis of the approximate formula (15). The largest deviations of the approximate temperature profiles from the exact ones take place at relatively small values of the parameter c. Our numerical investigations have shown that, only at $3/4 \le c \le 1$ and $m \le 1$, the approximate and exact temperature profiles are fairly close.



(**a**) (**b**)

Figure2. Temperature distribution for c = 1/5 (a) and c = 1/2 (b) for different parameter m: solid line – exact formula (10); dashed line – approximate formula (15)

Designating the fin efficiency by integration in Eq. (5b) by η and later inserting Eq. (13) into Eq. (7) yield the approximate expression [27]

$$\eta = 2c\frac{\mathrm{e}^{m\sqrt{2c(1+c)}}[2 - m\sqrt{2c(1+c)}] + \mathrm{e}^{m\sqrt{2\left(1+\frac{1}{c}\right)}}[2 + m\sqrt{2c(1+c)}] - 4\mathrm{e}^{m\frac{(1+c)^{3/2}}{\sqrt{2c}}}}{m^2(1-c)(1+c)^2[\mathrm{e}^{m\sqrt{2\left(1+\frac{1}{c}\right)}} + \mathrm{e}^{m\sqrt{2c(1+c)}}]} \tag{16}$$

whose evaluation can be done with a calculator. In contrast, the evaluation of the exact fin efficiency in Eq. (10) necessitates symbolic algebra codes, such as MAPLE, MATHEMATICA or MATLAB. Figure 3 shows the fin efficiency as a function of fin parameter m for $c = 0.1$, 0.3, 05 and 0.7. The relative error of calculating the parameter η is determined as

$$\varepsilon = \frac{\eta - \eta^*}{\eta^*}100\% \tag{17}$$

The calculations, performed by formula (16), have shown that the approximate estimation of the efficiency of the fin η is fairly coarse (Table 1). For example, at $c = 0.1$ and $m = 0.5$ and 2, we have an error $\varepsilon = 5.2$ and 14.8%, respectively. In the case where $c = 1/2$, the calculation error substantially decreases but remains fairly high. For example, at $m = 0.5$, we have $\varepsilon \approx 1\%$. The curves of the efficiency η calculated by the exact formula (10) and the approximate formula (16) are presented in Fig. 3. We see that, only at fairly large values of the parameter c ($0.5 \le c \le 1$), the known approximate solution (16) can be used with certain reserves. It should be also noted that the presence of the exponential function in the formula is inconvenient in the case where the engineering calculations of a hyperbolic-profile fin are performed with the use of a calculator.

Table 1. Fin efficiency η for a normalized radii ratio *c* = 0.1, 0.5 and variable fin parameter *m*

c	*m*	η^*	η	ε (%)
0.1	0.5	0.6576	0.6920	5.2
0.1	1	0.3526	0.3632	3.0
0.1	2	0.1552	0.1322	14.8
0.5	0.5	0.9647	0.9674	0.28
0.5	1	0.8743	0.8823	1.87
0.5	2	0.6497	0.6618	1.33

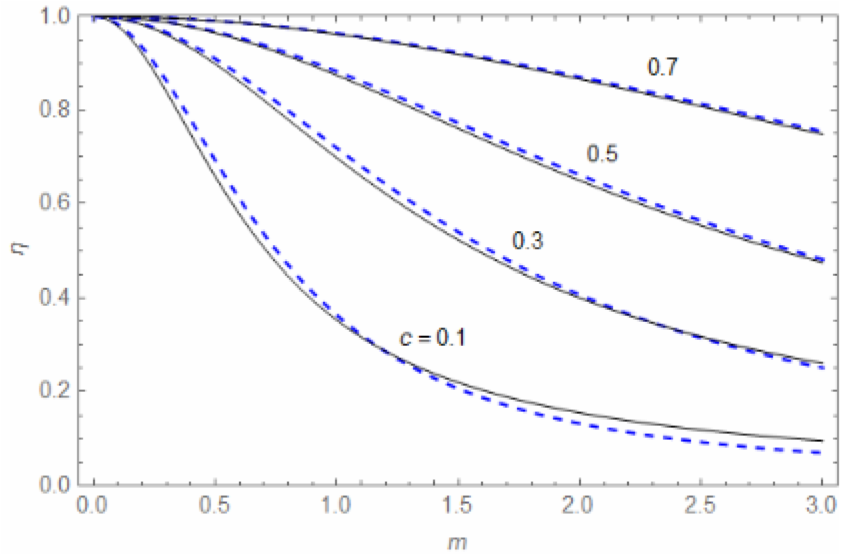


Figure 3. Fin efficiency as a function of the fin parameter *m* for different values of *c*: solid curve – exact solution; dashed curve – approximate solution (16)

3.2. Combined Integral Method (CIM)

The temperature function $\theta(R)$ is defined by the cubic polynomial

$$\theta(R) = 1 + A(r-c) + B(r-c)^2 + C(r-c)^3 \tag{18}$$

The coefficients *B* and *C* are determined with the use of the boundary condition (3) and the designation

$$\theta(1) = b \tag{19}$$

Solving the system of equations following from (3) and (19), we find the coefficients B and C and obtain

$$\theta(R) = 1 + A(r-c) + \left[3(b-1) - 2(1-c)A\right]\left(\frac{r-c}{1-c}\right)^2 - \left[3(b-1) - 2(1-c)A\right]\left(\frac{r-c}{1-c}\right)^3 \tag{20}$$

Fot determining the coefficients $A = \theta(c)/\mathrm{d}R$ and $b = \theta(1)$, we construct two integral relations. Integration of Eq. (1) in view of the boundary condition (3) and the relation $\theta(c)/\mathrm{d}R = A$ gives

$$\int_c^1 \theta(R) R\,\mathrm{d}R = -A/M^2 \tag{21}$$

Then we multiply the differential equation (1) by $(R-c)$ and integrate the relation obtained:

$$\int_c^1 \frac{\mathrm{d}\theta}{\mathrm{d}R^2}(R-c)\mathrm{d}R - M^2\int_c^1 \theta(R)R(R-c)\mathrm{d}r = 0 \tag{22}$$

Integrating the first termin (22) be parts two times, we arrive, in view of the boundary conditions (2) and (3) and the relations $\theta(1)=b$ and $\theta(c)/\mathrm{d}R=A$, at the integral relation

$$\int_c^1 \theta(R)R(R-c)\mathrm{d}R=(1-b)/M^2 \tag{23}$$

Substitution of the temperature function (20) into (21) and (23) gives the system of two linear algebraic equations

$$M^2(1-c)\left[9+21c+3(7+3c)b+(2+c-3c^2)A\right]=-60A \tag{24}$$

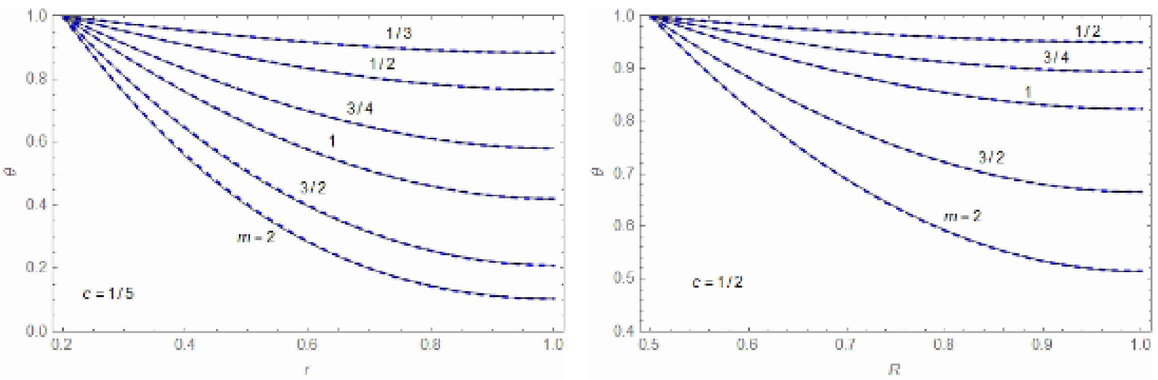
$$M^2(1-c)^2\left[4+5c+(16+5c)b+(1-c^2)A\right]=60(1-b) \tag{25}$$

Solving system (24), (25) , we obtain

$$A=\left.\frac{\mathrm{d}\theta}{\mathrm{d}R}\right|_{R=c}=-60M^2\frac{30(1-c^2)+(1-c)^3(1+4c+c^2)M^2}{3600+120(1-c)^2(9+4c)M^2+(1-c)^4(11+28c+6c^2)M^4} \tag{26}$$

$$b=\theta(1)=\frac{3600-120(1-c)^2(1+c)M^2+(1-c)^4(1+8c+6c^2)M^4}{3600+120(1-c)^2(9+4c)M^2+(1-c)^4(11+28c+6c^2)M^4} \tag{27}$$

The curves of the temperature function $\theta(R)$, calculated by formula (20) with the use of (26) and (27) at $c=1/5$ and $1/2$, are presented in Fig. 4. The temperature profiles constructed on the basis of (20) are almost coincident with the temperature profiles constructed on the basis of the exact formula (9). This allows the conclusion that the approximate solution of the boundary-value problem (1)–(3) on the basis of the CIM is wery good.



(**a**) (**b**)

Figure4. Temperature distribution for *c* = 1/5 (a) and *c* = 1/2 (b) for different parameter *m*: solid curve – exact formula (10); dashed curve – approximate formula (20)

For the temperature of the top of the fin, from (9) we obtain

$$\theta(1)=\sqrt{\frac{1}{c}}\,\frac{I_{1/3}\left(\frac{2m}{3\sqrt{c}}\right)I_{2/3}\left(\frac{2m}{3\sqrt{c}}\right)-I_{-1/3}\left(\frac{2m}{3\sqrt{c}}\right)I_{-2/3}\left(\frac{2m}{3\sqrt{c}}\right)}{I_{1/3}\left(\frac{2m}{3}c\right)I_{2/3}\left(\frac{2m}{3\sqrt{c}}\right)-I_{-1/3}\left(\frac{2m}{3}c\right)I_{-2/3}\left(\frac{2m}{3\sqrt{c}}\right)} \tag{28}$$

and arrive at the very simple formula

$$\theta(1)=\frac{3600c^2-120(1-c)^2(1+c)m^2c+(1-c)^4(1+8c+6c^2)m^4}{3600c^2+120(1-c)^2(9+4c)m^2c+(1-c)^4(11+28c+6c^2)m^4} \tag{29}$$

The curves of the temperature of the fin top $\theta(1)$ calculated by the approximate formula (29) are completely coincident with the exact ones calculated by formula (28) (Fig. 5).

Substitution of the derivative $\mathrm{d}\theta(c)/\mathrm{d}R$ determined by (26) into (6) gives a very simple formula for the efficiency of the fin

$$\eta=\frac{30+(1-c)^2(1+4c+c^2)\frac{m^2}{c(1+c)}}{30+(1-c)^2(9+4c)\frac{m^2}{c}+(1-c)^4(11+28c+6c^2)\frac{m^4}{120\,c^2}} \tag{30}$$

The curves of η calculated by the exact formula (10) and the approximate formula (30) are presented in Fig. 6. We state the practically complete coincidence of the indicated curves. The relative error in calculating the fin efficiency by formula (30) is wery small (Table 2). For example, at $c=0.1$ and $m=0.5$ and 2, we have an error $\varepsilon = 0.1$ and 1.1%, respectively. In the case where $c = 0.5$, at $m = 0.5$, we have an error $\varepsilon = 0.0003\%$. Consequetly, the simple quadrature formula (30) obtained by us allows one to perform an engineering calculation of the efficiency of hyperbolic-profile fins (with a very high accuracy) with the useofa usual calculator instead of the complex calculation by the exact formula (10) including modified Bessel functions of fractional order.

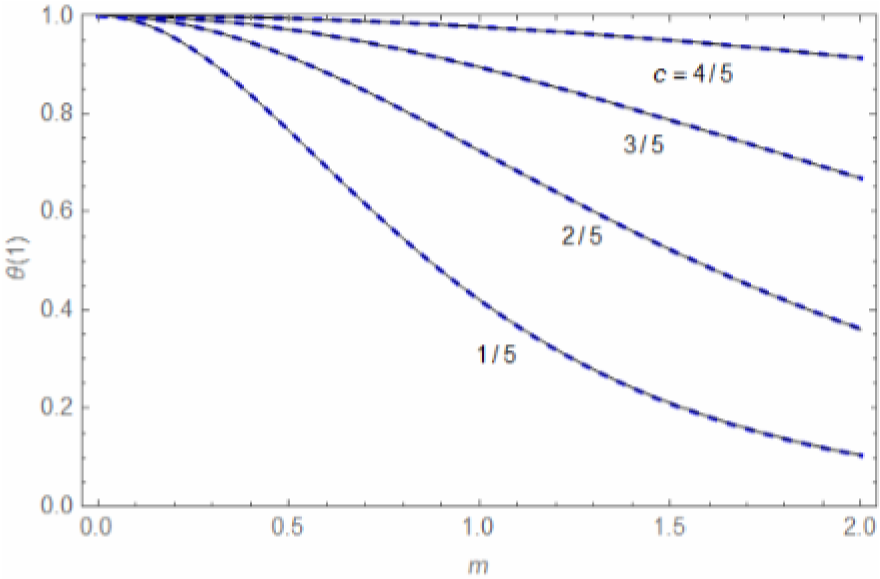


Figure5. Dimensionless tip temperature versus parameter m for different values of c

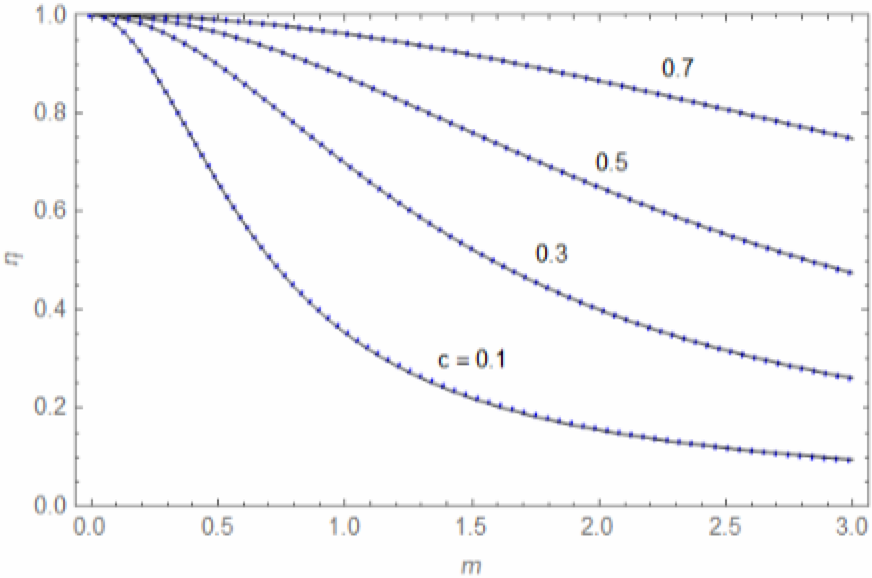


Figure6. Fin efficiency as a function of of fin parameter m for different values of c: solid curve –exact formula (10); dotted curve – approximate formula (30)

Table 2. Fin efficiency η for a normalized radii ratio c = 0.1, 0.5 and variable fin parameter m

c	m	η^*	η	ε (%)
0.1	0.5	0.6576	0.6584	0.1
0.1	1	0.3526	0.3551	0.7
0.1	2	0.1552	0.1569	1.1
0.5	0.5	0.9647	0.9647	0.0003
0.5	1	0.8743	0.8742	0.005
0.5	2	0.6497	0.6492	0.157

IV. CONCLUSION

The main conclusion to be drawn is that when dealing with annular fins of hyperbolic profile, usage of modified Bessel functions for solving the quasi-1D heat conduction equation can be obviated. Application of combined integrak method furnishes an unexpected, facile route that permits the determination of approximate analytical temperature distributions and heat transfer rates for engineering applications. It turns out that both the temperature distribution and the fin efficiency are of simple algebraic form. The two expressions can be evaluated with a calculator for real values of the two controlling parameters, the normalized radii ratio c and the parameter m. As compared to the known approximate solution based on the mean value theorem for integration, the computational formula obtained on the basis of the combined integral method is more simple and is substantially (by several orders of magnitude) more exact. Furthermore, the simple methodology can be easily extended to all annular fins with variable profiles: uniform, triangular, concave parabolic, convex parabolic and hyperbolic.

Conflict of interest

There is no conflict to disclose.

REFERENCES

[1]. Hewitt G.F. Shires G.L., Bott T.R. Process Heat Transfer, CRC Press, Boca Raton, FL., 1993.
[2]. Kraus A.D., Aziz A., Welty J.R., Extended Surface Heat Transfer, Wiley, NY, 2001.
[3]. Webb R.L. Principles of Enhanced Heat Transfer, Wiley Interscience, New York, NY, 1994.
[4]. Janna W. Engineering Heat Transfer, CRC Press, Boca Raton, FL, 2008.
[5]. Wolf. H., Heat Transfer, Harper & Row Limited, New York, 1983.
[6]. White F.M. Heat Transfer, Addison Wesley, Reading, MA, 1984.
[7]. Becker M., Heat Transfer: A Modern Approach, Kluwer Academic Publications, part of Springer-Verlag, Berlin, Germany, 1986.
[8]. Chapman A.J. Fundamentals of Heat Transfer, Fifth edition, Macmillan, NY, 1987.
[9]. Bayazitoglu Y. Elements of Heat Transfer, McGraw–Hill, NY, 1988.
[10]. Bejan A. Heat Transfer, John Wiley, NY, 1993.
[11]. Suryanarayana N.V. Engineering Heat Transfer, West Publishing Co., NY, 1995.
[12]. Arpaci V.S., Selamet A. Introduction to Heat Transfer, Prentice–Hall, Upper Saddle River, NJ, 2000.
[13]. Kreith F., Bohn M.S. Principles of Heat Transfer, Sixth edition, Brooks/Cole, Pacific Grove, CA, 2001.
[14]. Kaviany M. Principles of Heat Transfer, John Wiley, NY, 2001.
[15]. Çengel Y.A. Heat Transfer, Second edition, McGraw–Hill, NY, 2003.
[16]. Lienhard J.H., Lienhard V. A Heat Transfer Textbook, Phlogiston Press, Cambridge, MA, 2003.
[17]. Jiji L.M. Heat Transfer Essentials, Begell House, Redding, CT, 2004.
[18]. Holman J.P. Heat Transfer, Tenth edition, McGraw–Hill, NY, 2008.
[19]. Thomas L. Heat Transfer, Second edition, Capstone Co., Tulsa, OK, 2000.
[20]. Mills A.F. Basic Heat Transfer, Second edition, Prentice–Hall, Upper Saddle River, NJ, 1999.
[21]. Incropera F.P., DeWitt D.P., Bergman, T.L., Lavine, A.S. Introduction to Heat Transfer Fifth edition, John Wiley, NY, 2006.
[22]. Nellis G., Klein S. Heat Transfer, Cambridge University Press, NY, 2008.
[23]. Faghri A., Zhang Y., Howell J. Advanced Heat and Mass Transfer, Global Digital Press, Columbia, MO, 2010.
[24]. Han J.–C. Analytical Heat Transfer, CRC Press, Boca Raton, FL, 2012.
[25]. SchmidtE. Die Wa¨rmeu¨bertragung durch Rippen, Zeitschrift des Vereines Deutscher Ingenieure, 1926, Vol. 70, pp. 885-9 and 947-951.
[26]. Arauzo I., Campo A, and Cortés C. Quick estimate of the heat transfer characteristics of annular fins of hyperbolic profile with the power series method, Applied Thermal Engineer-ing, Vol. 25, pp. 623–634, 2005.
[27]. Campo A., Cui T. Temperature/Heat transfer analysis of annular fins of hyperbolic profile relying on the simple theory for straight fins of uniform profile, ASMW J. Heat Transfer, 2008,Vol. 130, No 5.
[28]. Campo A., Morrone B., Chikh S.Easy and rapid computation of the transfer of heat from annular fins of nearly optimal profile with the finite-difference technique and the shooting method, International Journal of Numerical Methods for Heat & Fluid Flow, 2004, Vol. 14 No. 8, pp. 1002-1010.
[29]. Campo A., Lira I. Simple approximate analytic treatment of annular fins of variable profile in a heat transfer course Latin American and Caribbean Journal of Engeneering Education, 2012, Vol. 6, No 1, pp. 32–38.
[30]. Yang Y.T, Chang C.C., Chen C.K. A double decomposition method for solving the annular hyperbolic profile fins with variable thermal conductivity, Heat Transf. Eng., 2010, 31, No 4, pp. 1165–1172.
[31]. Schneider P.J. Conduction Heat Transfer, Addison–Wesley, Reading, MA, 1955.
[32]. Abramowitz M., Stegun A. Handbook of Mathematical Functions, United States Government Printing Office, Washington, DC, 1964.