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Simplex C++ Syntax for Solving Chemical Engineering Cost Optimization Problems

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Abstract- *Chemical Engineering is a profession in which the knowledge of basic sciences and other natural sciences is applied to develop economic ways of using materials and energy for the benefit of mankind. Therefore, it is a core responsibility of chemical engineers to either maximize or minimize the cost of manufacturing useful products. This has to do with identifying constraints affecting variables for the purpose of optimization. Most often, engineers trust Simplex linear programming (LP) technique as it can solve multiple variable problems. Softwares readily available to do the tasks are Tora, Lingo and Excel Solver among others. C++ program based on Simplex technique will be run for solution of three literature work on chemical engineering optimization and result compared with the Lingo application. Results show that, values gotten in problem 1 (core binder production cost minimization), problem 2 (profit maximization for bakery production in Indonesia) and problem 3 (cost maximization for Coca Cola Bottling Company in Nigeria) corresponds with their respective literature values. It can be concluded that the C++ source code could also be an invaluable tool to solve linear programming problems (LPP).*

Keywords: *Simplex method, linear programming, C++ programming, Lingo, chemical engineering optimization*

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I. INTRODUCTION

Simplex Optimization Technique is one of several methods of solving linear programming (LP) or linear optimization problems. A linear programming problem (LPP) may be defined as the problem of maximizing or minimizing a linear function subject to linear constraints. The constraints may be equalities or inequalities [8]. The linear programming is the most popular mathematical technique which deals with the optimization of linear functions subject to linear constraints [14]. Any linear program consists of four parts: a set of decision variables, the parameters, the objective function, and a set of constraints [5]. The decision-maker's intention mostly is to control the decision variable which is normally restricted by constraints. The objective function is a mathematical expression that maximizes or minimizes for a particular combinations of decision variables.

Kaur, et al., (2021) hopes to address the frequent suicide by farmers in India caused by frustrations and loses they face on routine basis, using linear programming to help them know which crop and in how much quantity they should plant to earn optimum income. Apart from agriculture, the application of LP cut across various fields, as shown in Table 1:

Table 1: Applications of Linear Programming

	Fields/Sectors	Uses
1.	Airlines	Schedule flights, taking into accounts both scheduling aircraft and scheduling staff
2.	Manufacturing Companies	Maximize profit as well as plan and schedule production
3.	Healthcare Institutions	To efficiently identify a kidney donation chain and to time the supply of medicines and equipment before they run out.
4.	Logistics and transportation industry	Find the shortest path/route, travel time or pricing strategy
5.	Financial Institutions	Schedule payments, funds transfer between institutions and to determine the characteristics of the loan offer.
6.	Engineering	Solves design and manufacturing problems (helpful for doing shape optimization)
7.	Food Industry	Help nutritionists to plan dietary needs
8.	Retailers	Determine how to order and organize deliveries
9.	Energy Industry	Optimize the electric load, shortest distribution lines as well as the electrical power grid design
10.	Delivery Services	To maximize shipment time and reduce cost
11.	Agricultural Sector	Know the type and quantity of crops to be planted to increase revenue

This technique has also been applied in advertising, personnel and blending problems. Engineering consists of several branches. Table 1 only generalized the applications of Simplex LP techniques in engineering. Chemical engineering is one branch of engineering where this optimization techniques finds application. Examples of such application is the blending systems for chemical production that requires profit optimization. Also, Dragicevic, et al., (2009) explains how LP method can be used to minimize the total costs of energy utilized in steam condensing systems. Application of the Simplex Method of Linear Programming Model to Saclux Paint Production has also been studied by Okereke (2013). Such LP problems can be solved using methods in Figure 1:

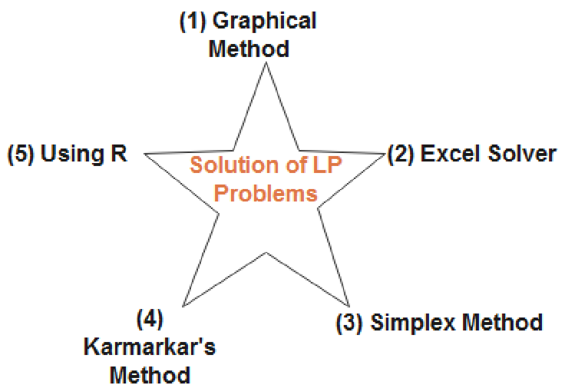


Fig. 1: Method of Solution for Linear Programming Problem

The graphical and Karmarkar’s method had been explained vividly by Vaidya, et al., (2020) and Akpan, et al., (2017) respectively. Excel Solver and R programming package as seen in Figure 1 are methods involving the use of computer applications. As the number of decision variables increase; graphical, Karmarkar and Simplex methods becomes difficult to work with manually. The fastest means of solving multiple variable problems is the use of computers. Apart from MS Excel, other softwares such as Lingo, Scilab and Tora can be applied.

II. EXPLAINING THE SIMLEX METHOD

The simplex method was developed by American mathematician George B. Dantzig in 1947 [12]. Steps involved are summarized in Table 2

Table 2: Solution Steps of the Simplex Method

(A) Form the Modified Problem ✓ If any problem constraints have negative constants on the right side, multiply both sides by -1 to obtain a constraint with a nonnegative constant. Remember to reverse the direction of the inequality if the constraint is an inequality ✓ Introduce a **slack variable** for each constraint of the form $\leq$. ✓ Introduce a **surplus variable** and **an artificial variable** in each $\geq$ constraint. Slack variables are additional variables that are introduced into the linear constraints of a linear program to transform them from inequality constraints to equality constraints ✓ Introduce an artificial variable in each = constraint ✓ For each artificial variable **a,** add $-$Ma to the objective function. Use the same constant M for all artificial variables
(B) Form the **preliminary simplex tableau** for the modified problem
(C) Use row operations to eliminate the M*s* in the bottom row of the preliminary simplex tableau in the columns corresponding to the artificial variables. The resulting tableau is the **initial simplex tableau**. For a system tableau to be considered an initial simplex tableau, it must satisfy the following two requirements: ✓ Each **basic variable** must correspond to a column in the tableau with exactly one nonzero element. The remaining variables are then selected as **non-basic variables** ✓ The basic solution found by setting the non-basic variables equal to zero is feasible
(D) Perform Pivot Operations The objective of pivoting is to make an element above or below a leading one into a zero. Pivoting is done on a 1. Although you do not have to pivot on a one, it is highly desirable. Now, selecting a pivot: ✓ Pick the column with the most zeros in it ✓ Use a row or column only once ✓ Pivot on a one if possible ✓ Pivot on the main diagonal ✓ Never pivot on a zero ✓ Never pivot on the right hand side Select a pivot column, which will be the column that contains the largest negative coefficient in the row containing the objective function.

Table 2 was culled out of Dass (1988), Mahto (2015) and Hussain, et al., (2019). The algorithm can further be summarized as seen in Figure 2:

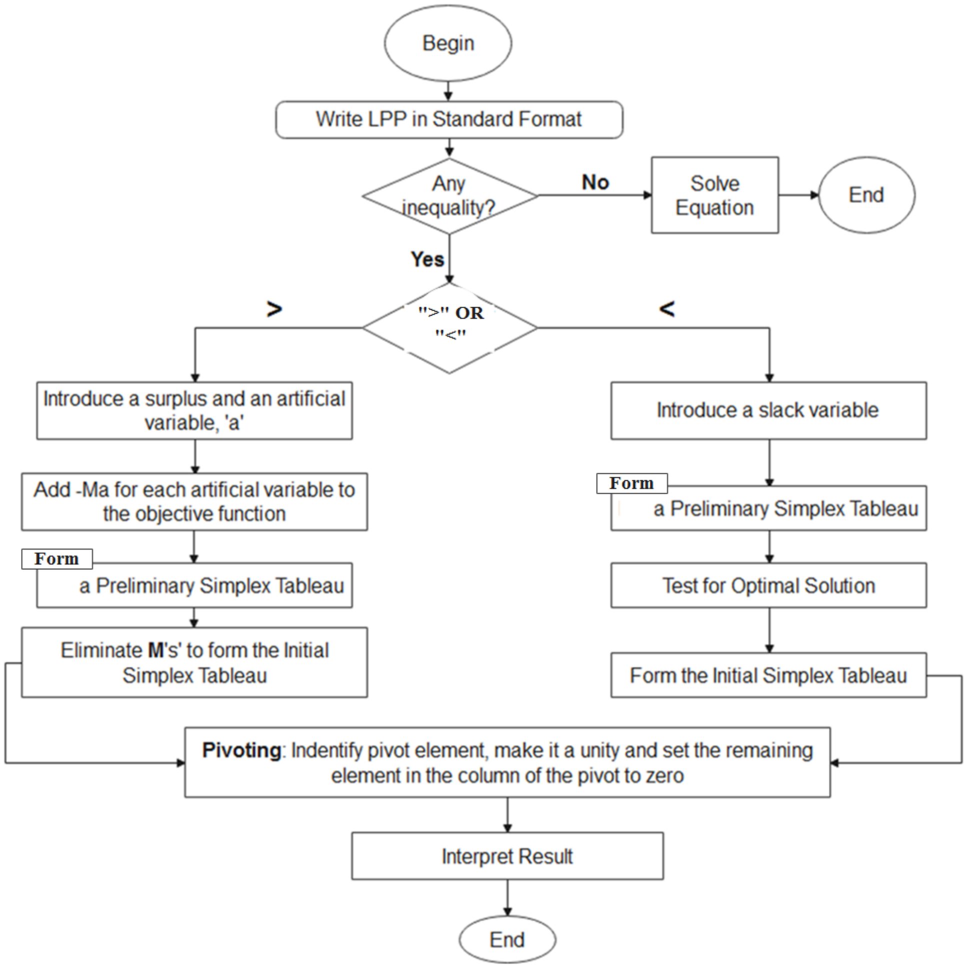


Fig. 2: Simplex Method Flowchart

C++, MATLAB and Mathematica are three most widely used programming applications in Chemical Engineering. Kapuno (2008) was able to write a book explaining both the C++ and MATLAB scope with regards to Chemical Engineering. Abubakar, et al., (2021), also wrote a comprehensive problem-solving C++ code for cubic equations of state. There are lots of codes to be written as far as chemical engineering is concerned.

III. COMPUTER SOLUTION

This work involves a test-run of the Simplex C++ source code for LPP, obtained from: http://www.cplusplus.com/forum/general/194307 (see Table 4, Appendix A) for the following chemical engineering problems:

Problem 1:

Ihom, et al., (2007) developed a core binder where Simplex Method of linear programming was used to determine the minimized cost of producing 1 kg of a core mixture. The objective function is: $C = 0.5x_1 + 21.8\,x_2 + 27.8\,x_3 + 1.59\,x_4$. Subject to

$x_1 = 100$
$x_2 = 5.0$
$x_3 = 1.5$
$x_4 = 2.0$

$$x_1\,,x_2\,,x_3\,,x_4 > 0$$

where $C =$ cost of producing 1 kg of core mixture (in Naira)

$x_1 =$ sand
$x_2 =$ manihot esculenta
$x_3 =$ cement
$x_4 =$ water

The result obtained by running the C++ syntax of Table 4 is as shown in Figure 3:

```
LINEAR PROGRAMMING

MAXIMIZE (Y/N) ? y

NUMBER OF VARIABLES OF ECONOMIC FUNCTION ? 4

NUMBER OF CONSTRAINTS ? 4

INPUT COEFFICIENTS OF ECONOMIC FUNCTION:
x1 ? 0.5
x2 ? 21.8
x3 ? 27.8
x4 ? 1.59
Right hand side ? 0

CONSTRAINT x1:
x1 ? 1
x2 ? 0
x3 ? 0
x4 ? 0
Right hand side ? 100

CONSTRAINT x2:
x1 ? 0
x2 ? 1
x3 ? 0
x4 ? 0
Right hand side ? 5.0
```

```
CONSTRAINT x3:
x1 ? 0
x2 ? 0
x3 ? 1
x4 ? 0
Right hand side ? 1.5

CONSTRAINT x4:
x1 ? 0
x2 ? 0
x3 ? 0
x4 ? 1
Right hand side ? 2.0


RESULTS:

VARIABLE x1: 100.000000
VARIABLE x2: 5.000000
VARIABLE x3: 1.500000
VARIABLE x4: 2.000000

ECONOMIC FUNCTION: 203.880000


--------------------------------
```

Fig. 3: Core Binder Production Cost Minimization Result

Clearly, the minimized cost of ₦ 203.88 ($0.5 – equivalent as of June 2021) of Figure 3 is the same result Ihom, et al., (2007) arrived at in page 160 of their work. When Problem 1 model is entered in Lingo 18.0 linear programming software (see Appendix B, Figure 5), the result proves to be the same.

Problem 2:

Another area of concern for chemical engineers is food processing. Anggoro, et al., (2019), formulated the LP model for a Bintang Bakery in Indonesia to maximize profit.

Table 3: Bintang Bakery Linear Optimization Model

Decision variables	x_1 = Bintang Bakery flavor (3640 packs) x_2 = Bintang Bakery mattress (1300 packs) x_3 = Bintang Bakery bargain (520 packs)
Constraints	Flour = $28x_1 + 100x_2 + 250x_3 \leq 400.000$ Sugar = $7x_1 + 25x_2 + 62x_3 \leq 250.000$ Developer = $1x_1 + 9x_2 + 4x_3 \leq 90000$ Softener = $1x_1 + 6x_2 + 2x_3 \leq 40000$ Yellow butter = $5x_1 + 20x_2 + 50x_3 \leq 90000$ Salt = $1x_1 + 3x_2 \leq 10000$ Milk powder = $1x_1 + 3x_2 + 2x_3 \leq 60000$ Liquid milk = $5x_1 + 20x_2 \leq 60000$ BOS butter = $5x_1 + 20x_2 \leq 90000$ Egg = $4x_1 + 15x_2 + 25x_3 \leq 70000$ Feeling = $14x_1 + 20x_2 \leq 200000$ White butter = $5x_3 \leq 90000$ Calcium = $2x_3 \leq 20000$ Production machine = $32x_1 + 132x_2 + 336x_3 \leq 475200$ Labor = $65x_1 + 209x_2 + 450x_3 \leq 748800$ $x_1 \geq 3640$ $x_2 \geq 1300$ $x_3 \geq 520$
Objective Function	$Z = 2500x_1 + 6000x_2 + 5000x_3$

There are a total of 18 constraints and 3 decision variables in Table 3. Excel Solver and C++ Simplex Source Code will give; $x_1 = 14.2857$, $x_2 = 0$ and $x_3 = 0$ which is an infeasible solution at $Z = 35714.3$. This is not the optimal solution as affirmed by Anggoro, et al., (2019). Lingo's solution is not different from the forgone.

Problem 3:

Akpan, et al., (2017) applied both Karmarkar and Simplex methods to maximize profit for the Nigerian Bottling Company (NBC) Port Harcourt. An optimal profit of ₦107,666,640.00 was obtained when the model was run using the Tora Software. From Figure 5, it can be deduced that the maximum profit is exactly the same with the result obtained by Akpan, et.al., (2017):

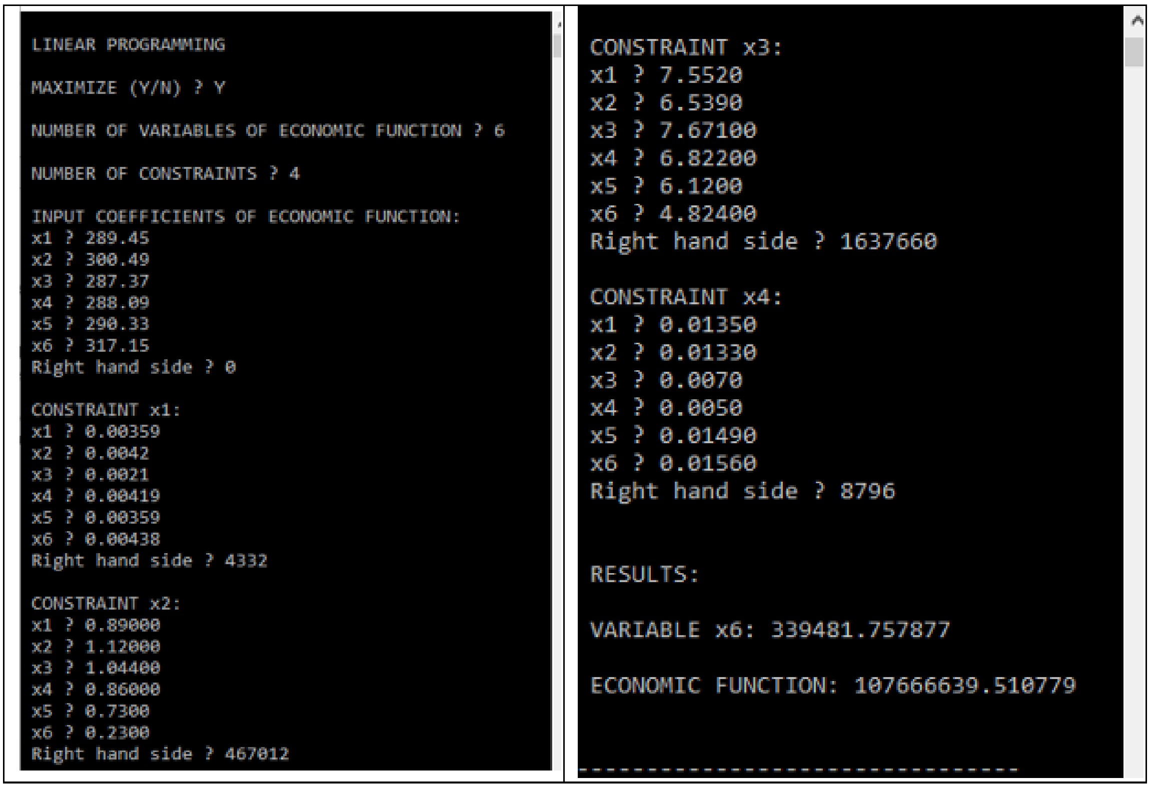


Fig. 4: Profit Maximization for the NBC, Port Harcourt

Where x_1 = Coke 50cl; x_2 = Coke 35cl; x_3 = Fanta 50cl; x_4 = Fanta 35cl; x_5 = Sprite 35cl and; x_6 = Schweppes 33cl. Also, $x_6 = 339482$ is the number of crates of Schweppes that will result to a maximum profit of ₦107,666,640 without considering other products. The Lingo Software could also be tested to verify the above result (Appendix B).

IV. CONCLUSION

Three chemical engineering LP or optimization problems have been considered for solution using C++ programming as well as the Lingo Software. These problems were drawn from published articles. The results were able to verify the correctness of values obtained in those article which further affirms the validity of the C++ Source Code. Surely, this source code can be adopted for solving not only chemical engineering linear optimization problems, but almost any LPP encountered. It is suitable for multiple variables and many constraints (as in Problem 2 having 18 constraint and Problem 3 having 6 variables). The use of other computer programming softwares (e.g. Java, R, Python and Fortran) is hereby recommended for solution to chemical engineering LPP.

ACKNOWLEDGEMENT

It is important to acknowledge Mr. Okoh Chibuike Francis of the University of Port Harcourt (Nigeria) for sharing his programming ideas; given that he is good at programming with Java, XML and JSON which is somewhat similar to C++. The coder of the simplex C++ syntax obtained in cplusplus.com, together with other co-authors are recognized for their commitment towards a successful completion of this paper.

APPENDIX

Appendix A

Problems 1, 2 and 3 are chemical engineering problems developed and solved by different authors in the field. Table 4 is the C++ coding mimicking the Simplex solution steps. They provide the results seen in Figure 3 and Figure 4.

Table 4: C++ Code for Simplex Method Linear Programming Problem

Simplex LP.cpp

```
#include <stdio.h>
#include <math.h>

#define CMAX 10 //max. number of variables in economic function
#define VMAX 10 //max. number of constraints

int NC, NV, NOPTIMAL,P1,P2,XERR;
double TS[CMAX][VMAX];
void Data()
{
    double R1,R2;
    char R;
    int I,J;

    printf("\n LINEAR PROGRAMMING\n\n");
    printf(" MAXIMIZE (Y/N) ? ");
    scanf("%c", &R);
    printf("\n NUMBER OF VARIABLES OF ECONOMIC FUNCTION ? ");
    scanf("%d", &NV);
    printf("\n NUMBER OF CONSTRAINTS ? ");
    scanf("%d", &NC);

    if (R=='Y' || R=='y')
        R1 = 1.0;
    else
        R1 = -1.0;
    printf("\n INPUT COEFFICIENTS OF ECONOMIC FUNCTION:\n");

    for (J = 1; J<=NV; J++)
    {
        printf(" x%d ? ", J);
        scanf("%lf", &R2);
        TS[1][J+1]=R2*R1;
    }

    printf(" Right hand side ? ");
    scanf("%lf", &R2);
    TS[1][1] = R2 * R1;

    for (I = 1; I<=NC; I++)
    {
        printf("\n CONSTRAINT x%d:\n", I);

        for (J = 1; J<=NV; J++)
        {
            printf(" x%d ? ", J);
            scanf("%lf", &R2);
            TS[I + 1][J + 1] = -R2;
        }
    printf(" Right hand side ? ");
    scanf("%lf", &TS[I+1][1]);
    }

    printf("\n\n RESULTS:\n\n");
    for(J=1; J<=NV; J++) TS[0][J+1] = J;
    for(I=NV+1; I<=NV+NC; I++) TS[I-NV+1][0] = I;
}

void Pivot();
void Formula();
void Optimize();
void Simplex()
{
    e10: Pivot();
    Formula();
    Optimize();

    if (NOPTIMAL == 1)
    goto e10;
}
```

```
void Pivot()
{
    double RAP,V,XMAX;
    int I,J;
    XMAX = 0.0;

    for(J=2; J<=NV+1; J++)
    {
        if (TS[1][J] > 0.0 && TS[1][J] > XMAX)
        {
        XMAX = TS[1][J];
        P2 = J;
        }
    }
        RAP = 999999.0;

    for (I=2; I<=NC+1; I++)
    {
        if (TS[I][P2] >= 0.0)
        goto e10;
        V = fabs(TS[I][1] / TS[I][P2]);

        if (V < RAP)
        {
            RAP = V;
            P1 = I;
        }
    e10:;
    }
        V = TS[0][P2]; TS[0][P2] = TS[P1][0]; TS[P1][0] = V;
}

void Formula()
{;
    //Labels: e60,e70,e100,e110;
    int I,J;

    for (I=1; I<=NC+1; I++)
    {
        if (I == P1)
        goto e70;

        for (J=1; J<=NV+1; J++)
        {
            if (J == P2)
            goto e60;
            TS[I][J] -= TS[P1][J] * TS[I][P2] / TS[P1][P2];
        e60:;
        }
    e70:;
    }
        TS[P1][P2] = 1.0 / TS[P1][P2];
    for (J=1; J<=NV+1; J++)
    {
        if (J == P2)
        goto e100;
        TS[P1][J] *= fabs(TS[P1][P2]);
    e100:;
    }

    for (I=1; I<=NC+1; I++)
    {
        if (I == P1)
        goto e110;
        TS[I][P2] *= TS[P1][P2];
    e110:;
    }
}

void Optimize()
{
    int I,J;
    for (I=2; I<=NC+1; I++)
        if (TS[I][1] < 0.0) XERR = 1;
            NOPTIMAL = 0;
        if (XERR == 1) return;
    for (J=2; J<=NV+1; J++)
        if (TS[1][J] > 0.0) NOPTIMAL = 1;
}

void Results()
{
//Labels: e30,e70,e100;
    int I,J;
    if (XERR == 0)
    goto e30;
        printf(" NO SOLUTION.\n");
    goto e100;

    e30:
    for (I=1; I<=NV; I++)
        for (J=2; J<=NC+1; J++)
        {
            if (TS[J][0] != 1.0*I)
            goto e70;
            printf(" VARIABLE x%d: %f\n", I, TS[J][1]);
            e70: ;
        }

    printf("\n ECONOMIC FUNCTION: %f\n", TS[1][1]);
    e100:
    printf("\n");
}

int main()
{
    Data();
    Simplex();
    Results();
}               //end of file simplex.cpp
```

LIST OF MAIN VARIABLES:

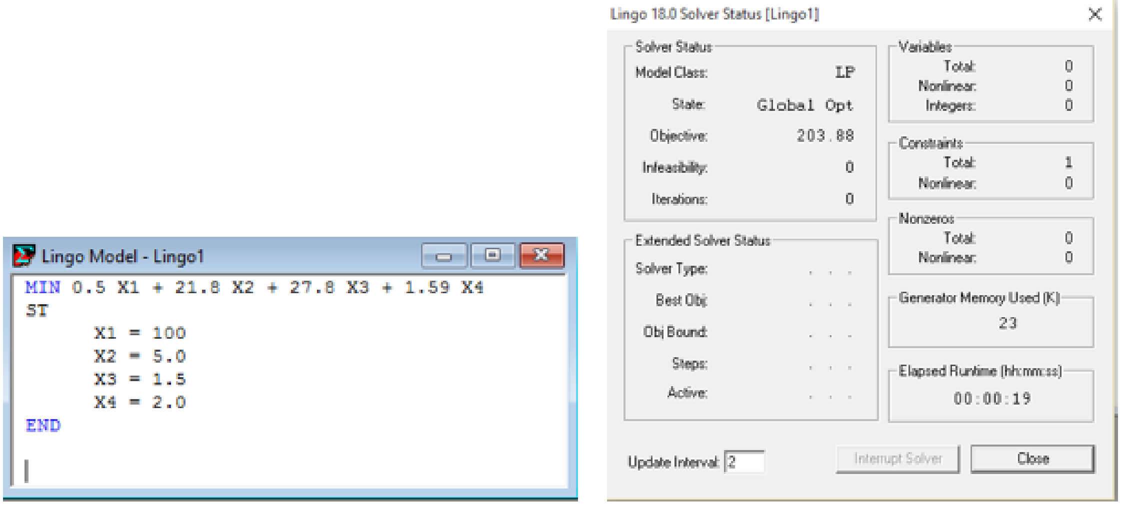
R:	MAXIMIZE = Y, MINIMIZE = N
NV:	NUMBER OF VARIABLES OF ECONOMIC FUNCTION (TO MAXIMIZE OR MINIMIZE).
NC:	NUMBER OF CONSTRAINTS
TS:	SIMPLEX TABLE OF SIZE NC+1 x NV+1
R1:	=1 TO MAXIMIZE, =-1 TO MINIMIZE
R2:	AUXILIARY VARIABLE FOR INPUTS
NOPTIMAL	BOOLEAN IF FALSE, CONTINUE ITERATIONS
XMAX:	STORES GREATER COEFFICIENT OF ECONOMIC FUNCTION.
RAP	STORES SMALLEST RATIO > 0
V:	AUXILIARY VARIABLE
P1,P2:	LINE, COLUMN INDEX OF PIVOT
XERR:	BOOLEAN IF TRUE, NO SOLUTION.

Source: (http://www.cplusplus.com/forum/general/194307)

The last row of Table 4 tells us that C++ sets ‘Y’ as the maximize option and ‘N’, the minimize function.

Appendix B

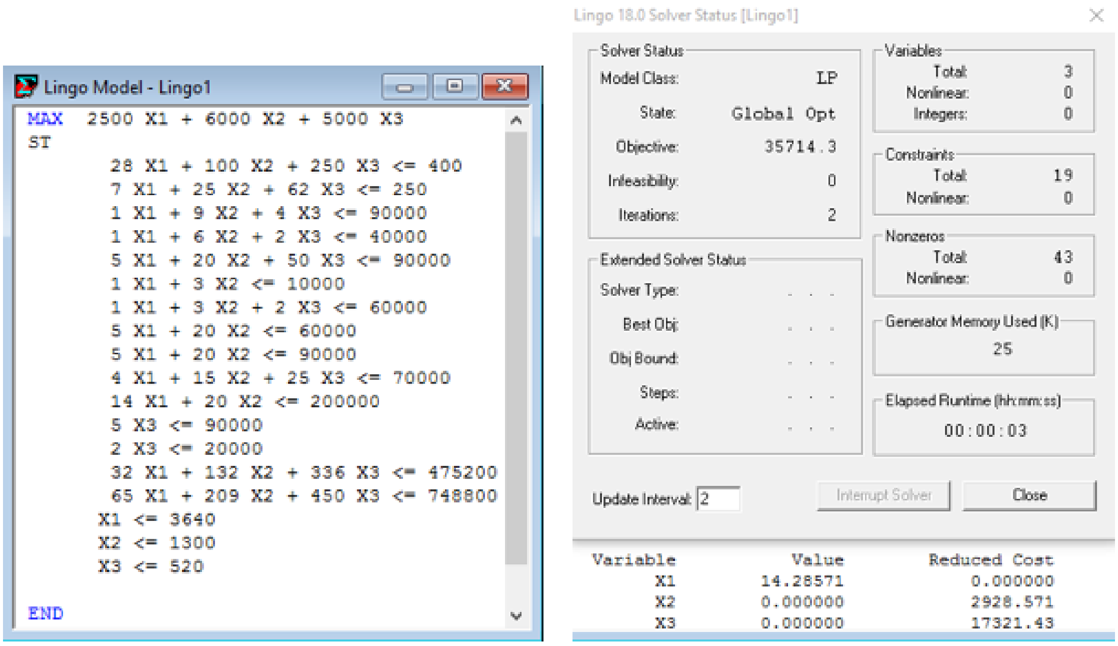
The syntax of typing any LPP in Lingo software is as shown in Figure 5, 6 and 7.



(a) Lingo (Problem 1) Model (b) Lingo Solver Output

Fig. 5: Problem 1 Lingo Solver Result

This however proves Figure 3. Solution to Problem 2 is infeasible. The setup that agrees with Excel Solver and C++ result is displayed in Figure 6:



(a) Lindo (Problem 2) Model (b) Lingo Solver Output

Fig. 6: Problem 2 Lingo Solver Result

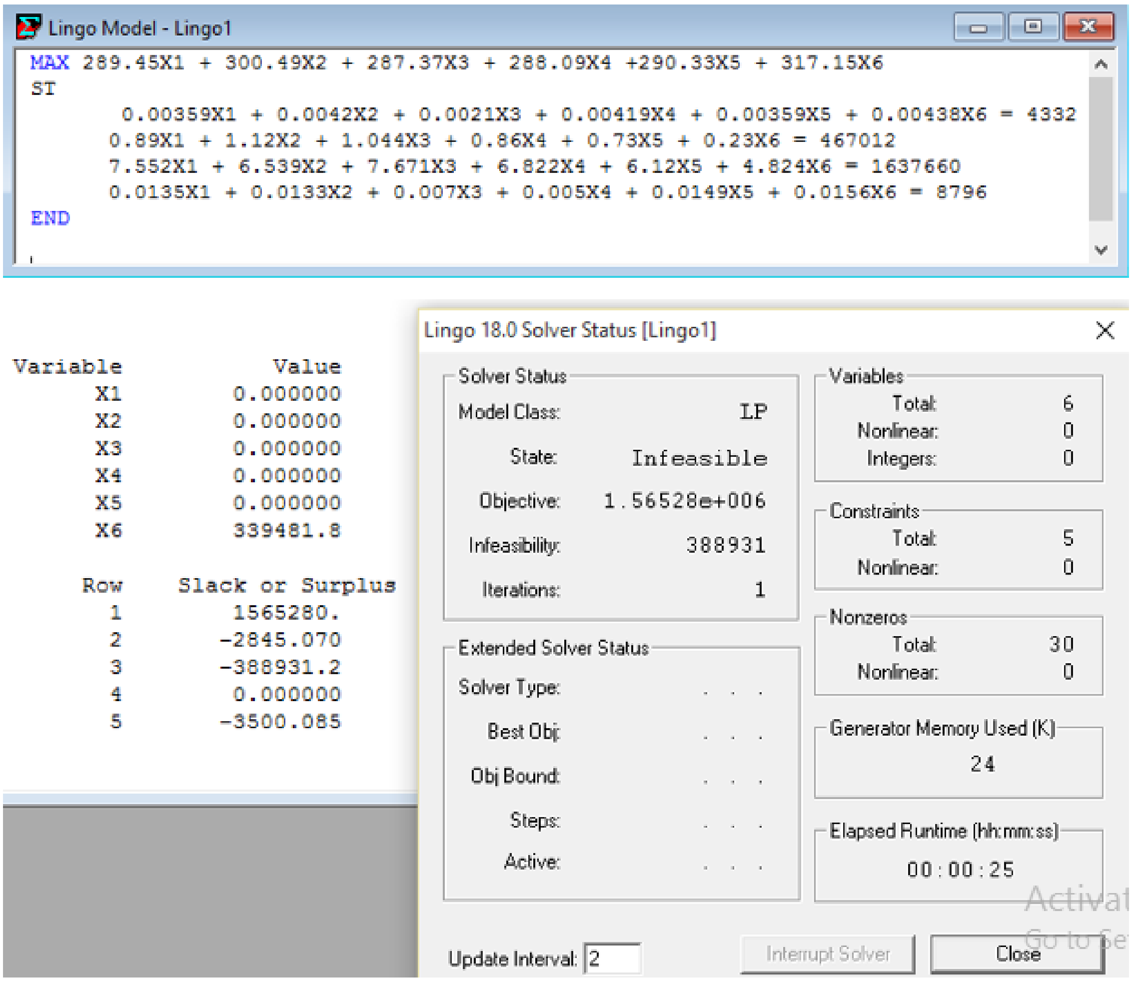


Fig. 7: Problem 3 Lingo Solver Result

Though Lingo Solver claims infeasibility in problem 3 model, crates of Schweppes 33cl, $x_6 = 339481.8$ (which is correct). All other variables are zero, therefore the objective function $= 317.15\,x_6 = 107666652.9$ (also the right result). As of June 19, 2021, N107,666,652.9 was equivalent to $262,248.4 which is the expected maximum profit for the company.

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