Research Inventy: International Journal of Engineering And Science
Vol.8, Issue 3 (July 2021), PP -06-13
Issn (e): 2278-4721, Issn (p):2319-6483, www.researchinventy.com

PID Algorithm for Trajectory Tracking Control of Two-Link Robot Manipulator

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Abstract: *In this paper, we suggest a structure of PID controller for trajectory tracking of two-link robot manipulator. Because PID algorithm is not guaranteed by mathematics, genetic algorithm is used to find the suitable control parameters. The ability of controller is proved through Matlab/Simulink simulation.*
Keywords: *PID control, genetic algorithm, robot manipulator, two-link.*

Date Of Submission: xx-xx-xxxx | Date Of Acceptance: xx-xx-xxxx

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I. INTRODUCTION

Manipulator is a popular model for control researching [2], [3]. Its simple mechanical structure and nonlinear mathematical model are suitable for trajectory tracking algorithm study. A two-link structure is enough for this robot to work in a space. In many flexible control method, PID is still a efficient and simple method. In industry, mostly people use PID as main controller for their purpose [4]. Thence, a research about two-link robot manipulator is necessary as a reference for other later research to be followed.

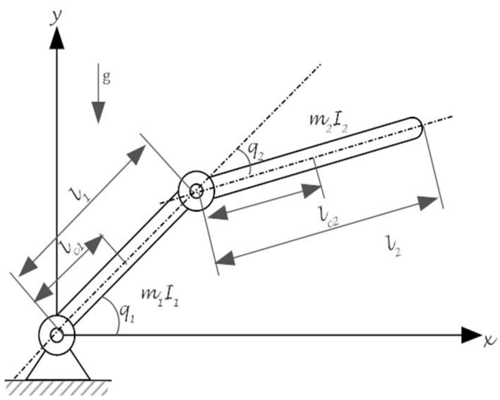

Figure 1. Structure of two-linked manipulator

Sympol	Description	Value	Unit
m_1	Link-1 mass	4	kg
m_2	Link-2 mass	3	kg
l_i	Link length	1	m
l_{Ci}	A scalar from the i-th centroid of the link to the joint of the next link	0.5	m
g	Gravity acceleration	9.81	m/s^2
I_1	Moment of inertia of link-1	0.333	$kg.m^2$
I_2	Moment of inertia of link-2	0.25	$kg.m^2$

Table 1. System Parameter

II. KINEMATICS

By geometric analysis, the position of the end-effector is writen:

$$P_{EE} = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} l_1 \cos q_1 + l_2 \cos(q_1 + q_2) \\ l_1 \sin q_1 + l_2 \sin(q_1 + q_2) \end{bmatrix} \quad (1)$$

(1) is Forward Kinematic formula of two-link robot manipulator.
From (1):

$$\begin{cases} l_2 \cos(q_1 + q_2) = x - l_1 \cos q_1 \\ l_2 \sin(q_1 + q_2) = y - l_1 \sin q_1 \end{cases}$$

$$\rightarrow (l_2 \cos(q_1 + q_2))^2 + (l_2 \sin(q_1 + q_2))^2 = (x - l_1 \cos q_1)^2 + (y - l_1 \sin q_1)^2$$

$$\rightarrow (2x l_1) \cos q_1 + (2y l_1) \sin q_1 = x^2 + y^2 + {l_1}^2 - {l_2}^2 \quad (2)$$

(2) can be rewriten as: $a \cos q_1 + b \sin q_1 = d$
where:

$$\begin{cases} a = 2xl_1 \\ b = 2yl_1 \\ d = x^2 + y^2 + {l_1}^2 - {l_2}^2 \end{cases}$$

$$\rightarrow \frac{a\cos q_1}{\sqrt{a^2+b^2}} + \frac{b\sin q_1}{\sqrt{a^2+b^2}} = \frac{d}{\sqrt{a^2+b^2}} \tag{3}$$

(2) can be rewrite as: $\cos q_1 \cos\alpha + \sin q_1 \sin\alpha = \frac{d}{\sqrt{a^2+b^2}}$

$$\rightarrow \begin{cases} \cos(q_1 - \alpha) = \dfrac{d}{\sqrt{a^2+b^2}} \\ \sin(q_1 - \alpha) = \pm\sqrt{1 - \dfrac{d^2}{\sqrt{a^2+b^2}}} \end{cases}$$

$$\rightarrow \begin{cases} q_1 = \alpha + atan2\left(\pm\sqrt{1 - \dfrac{d^2}{\sqrt{a^2+b^2}}}, \dfrac{d}{\sqrt{a^2+b^2}}\right) \\ q_2 = atan2(y - l_1\sin q_1, x - l_1\cos q_1) \end{cases} \tag{4}$$

(4) is the Inverse Kinematic formula of two-link robot manipulator.

III. DYNAMIC EQUATIONS

From [1], dynamic equations of two-link robot manipulator are obtained from Euler-Lagrange method as:

$$M(q)\ddot{q} + V(q,\dot{q})\dot{q} + G(q) = \tau \tag{5}$$

where:

$$M(q) = \begin{bmatrix} l_2^2 m_2 + 2l_1l_2m_2\cos\theta_2 + l_1^2(m_1+m_2) & l_1l_2m_2\cos\theta_2 \\ l_2^2m_2 + l_1l_2m_2\cos\theta_2 & l_2^2m_2 \end{bmatrix}$$

$$V(q,\dot{q}) = \begin{bmatrix} -m_2l_1l_2\dot{\theta}_2^2\sin\theta_2 - 2m_2l_1l_2\dot{\theta}_1\dot{\theta}_2\sin\theta_2 \\ m_2l_1l_2\dot{\theta}_1^2\sin\theta_2 \end{bmatrix}$$

$$G(q) = \begin{bmatrix} m_2l_2g\cos(\theta_1+\theta_2) + (m_1+m_2)l_1g\cos\theta_1 \\ m_2l_2g\cos(\theta_1+\theta_2) \end{bmatrix}$$

IV. PID STRUCTURE

1. Controller design

A proportional–integral–derivative controller (PID controller) is a close-loop which is popular in industry, specially automation controlling. A PID controller is normally built with three distinct components, namely proportional, integral, and derivative, where: proportional term defines current value of error, integral term accounts for past values of the error and integrates them over time, derivative term is a estimation of the trend of errors based on its current rate of change.

A PID controller calculates error value as the difference between a desires setpoint and measured process variable, and then applies the correct value based on three term above as an output of this controller.

In this paper, the input r will calculated via Inverse Kinematic and the output y is the actual joint position of the system obtained by dynamic analysis. The input variable of controller concludes error e between the desired joint position and the actual joint position. τ is the joint torque, also is the output variable of PID controller.

The PID controller structure is shown in *Figure 2*. The system controller structure is shown in *Figure 3*.

From [1], the control torques are computed as $\tau = k_p e + k_i \int e\, dt + k_d \dot{e} + G(q)$

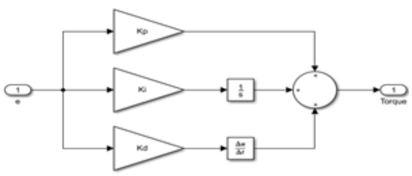

Figure 2. PID controller structure

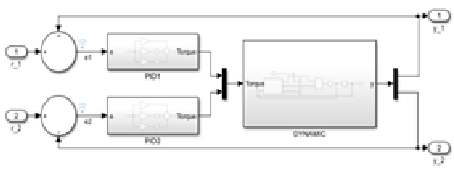

Figure 3. Structure of PID controllers for robot manipulator

2. Tracjector design

Simulations are carried out on a two-link robot manipulator as described in *Fig.1*. The system parameters are shown in *Table.1*. The desired trajectory of the end-effector position is a circle with the equations are shown in (6). The trajectory parameters are chosen as in *Table 2*. Time taken is 10s. Sample time is 0.01s.

$$\begin{cases} x_{set}(t) = A_x \cos\left(\frac{\pi t}{T}\right) + a \\ y_{set}(t) = A_y \sin\left(\frac{\pi t}{T}\right) + b \end{cases} \quad (6)$$

Symbol	Description	Parameter	Unit
A_x	X-axis amplitude	0.28	m
A_y	Y-axis amplitude	0.28	m
T	Period	5	s
a	X-axis position coordinate	0.85	
b	Y-axis position coordinate	0	

Table 2. Trajectory Parameter

V. SIMULATION

A Trajectory tracking controlling structure is built as in *Fig. 4*. We can see that, after providing the desired trajectory, the torque is controlled by inverse kinematics of the manipulator, and then calculated via dynamic analysis and forwarn kinematics. As a result, the manipulator move in the correct trajectory, parallel to this process is feedbacking to find the error values.

Choosing parameters for PID controller as: $K_p = 250, K_i = 1, K_d = 50$. The result is displayed in *Fig. 5*, *Fig. 7* and *Fig. 8*. The error values of joint 1 and joint 2 can be seen in *Fig. 6*. The average error value of joint 1 is $4.71 \times 10^{-4}\ rad$. The avarage error value of joint 2 is $1.99 \times 10^{-5}\ rad$. Those figures are very small compared to the output of the whole simulating process, so they could be adopted.

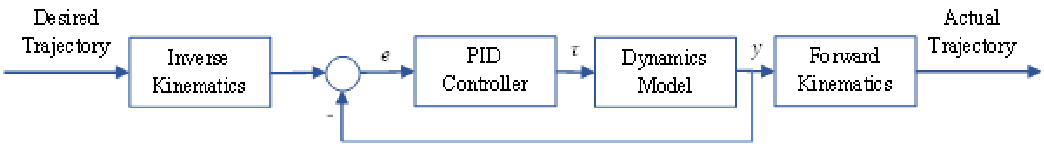

Figure 4. Trajectory tracking controlling structure

Simulation results are shown in Figure 5 to Figure 8. The motion of the robot arm follows well the round-trajectory in Figure 5. Errors between the angles of links and the reference trajectories are shown in Figure 6. We see that these errors are moving to zero as the movement of these links. Values of links are compared with the references in Figure 7, 8. They are very closed to each other. Thence, the PID controllers are successful in controling the system.

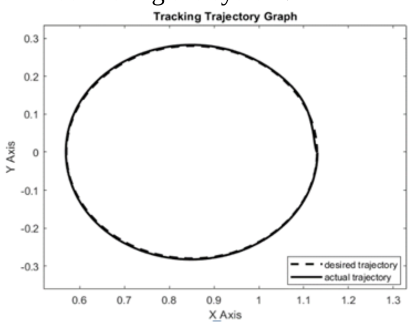

Figure 5. Tracking trajectory graph

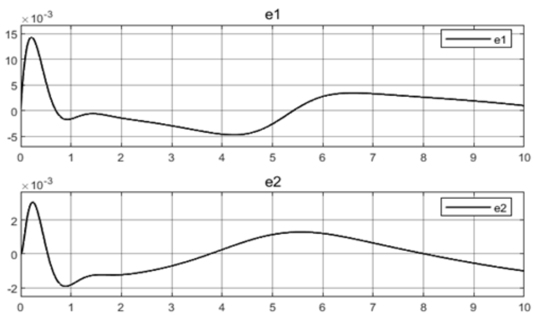

Figure 6. Errors of joint 1 and joint 2

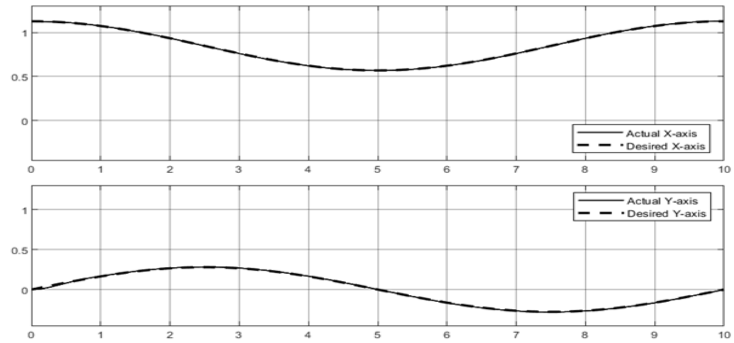

Figure 7. Tracking desired angles

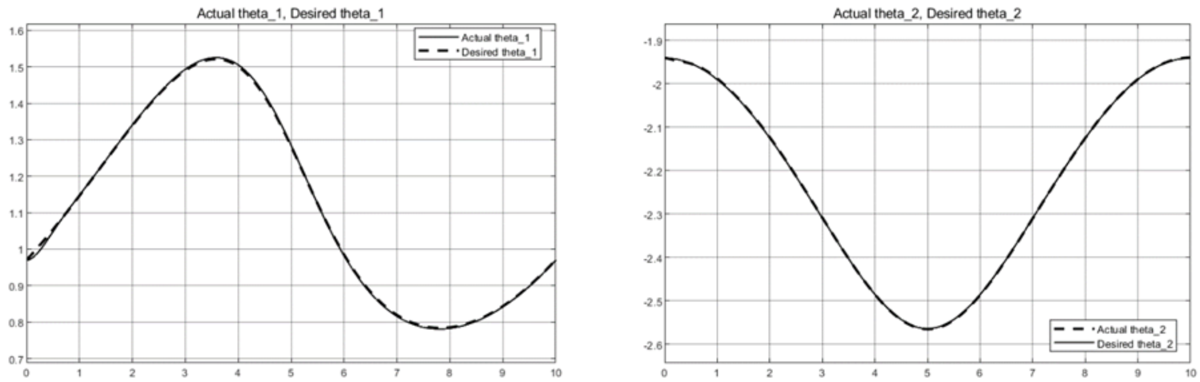

Figure 8. Tracking desired coordinate

VI. CONCLUSION

In this paper, authors presented the modeling, analysic, calculation and simulation of a two-dof robot manipulator. We propose a structure of PID controllers to control this model to track the round-traejctory. From simuation results, we see that this controller works well and make the system to follw the traejctory. Thence, PID controller can control the manipulator tranking the circle trajectory successfully.

Conflict of interest
There is no conflict to disclose.

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